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AFAPL-TR-65-45
Part VI

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY

Part VI: The Influence of Electromagnetic Forces on the
Stability and Response of an Alternator Rotor.

J. Lund
T. Chiang

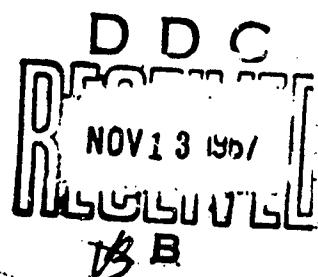
Mechanical Technology Incorporated

TECHNICAL REPORT AFAPL-TR-65-45, PART VI

September 1967

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AFAPL-TR-65-45

Part VI

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY

**Part VI: The Influence of Electromagnetic Forces on the
Stability and Response of an Alternator Rotor.**

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**J. Lund
T. Chiang**

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**Air Force Aero Propulsion Laboratory
Directorate of Laboratories
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio**

FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110 under USAF Contract No. AF 33(615)-3238. ^{new} The contract was initiated under Project No. 3044, Task No. 304402. The work was administered under the direction of the Air Force Aero Propulsion Laboratory, Research and Technology Division, with Mr. Michael R. Chasman (APFL) acting as project engineer.

This report covers work conducted from 1 February 1966 to 1 May 1967.

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This report is Part VI of final documentation issued in multiple parts.

This technical report has been reviewed and is approved.

Arthur V. Churchill
ARTHUR V. CHURCHILL, Chief
Fuels, Lubrication and Hazards Branch
Support Technology Division
Air Force Aero Propulsion Laboratory

ABSTRACT

This volume presents an analytical investigation of the vibrations induced in an alternator rotor by the generated electromagnetic forces. Formulas are given from which the magnetic forces can be calculated for three brushless alternator types: 1) the homopolar generator, 2) the heteropolar inductor generator, and 3) the two-coil Lundell generator. Numerical examples are given to illustrate the practical use of the formulas.

Two computer programs have been written for evaluation of the effect of the magnetic forces on the rotor. Manuals are provided for both programs, containing listings of the programs and giving detailed instructions for preparation of input data and for performing the calculations. The first computer program examines the stability of the rotor and the second program calculates the amplitude response of the rotor resulting from a built-in eccentricity and misalignment between the axes of the rotor and the alternator stator. Sample calculations are provided.

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SYMBOLS

A	Cross-sectional area of a shaft section, inch ²
A	Area of a pole, inch ²
A_N, A_S	Area of a north pole or a south pole, inch ²
A_T	Area of a stator tooth in the heteropolar generator, inch ²
A, B	Influence matrices for rotor, eq. (H.62), Appendix H
a	$-\frac{AB_4}{72C} L_p$, magnetic force gradient for 4 pole homopolar generator, lbs.
$a_{1n}, a_{2n}, \dots, a_{10n}$	Influence coefficients for shaft section n, eq. (H.46)
B	Combined damping of rotor bearings, lb-sec/inch
B_o	Average flux density, Lines/inch ² or Kilolines/inch ²
B_N, B_S	Flux density at north poles or south poles, Lines/inch ²
$B_{xx}, B_{xy}, B_{yx}, B_{yy}$	Damping coefficients of bearing, lbs-sec/inch
c	Radial airgap or clearance, inch
E	Youngs modulus, lbs/inch ²
E_A	Effective or voltmeter value of line voltage, volts
E_f	Voltage of field coil, volts
E_k	Rotor impedance matrix at k'th harmonic
e	Eccentricity between rotor center and stator center, inch
F	$\vec{F}_{\text{ector}} = \begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix}$
F_{Nx}, F_{Ny}	x and y-component of magnetic force due to north poles, lbs.
F_{Sx}, F_{Sy}	x and y-component of magnetic force due to south poles, lbs.
F_x, F_y	x and y-component of total magnetic force in centerplane of alternator, lbs.
\mathcal{J}	Magnetomotive force
\mathcal{J}_A	Magnetomotive force of armature reaction
$\mathcal{J}_{Ad}, \mathcal{J}_{Aq}$	Demagnetizing and crossmagnetizing component of armature reaction
\mathcal{J}_f	Magnetomotive force of field coil
$\mathcal{J}_N, \mathcal{J}_S$	Drop in magnetomotive force across the north or the south poles

f	Drop in magnetomotive force across airgap in heteropolar generator
f_0	$= 1/2 \frac{N_f}{R_f} E_f$
f_d	$= 2 \frac{ B_{Ad} }{ B_f }, \text{ eq. (E.2), Appendix V}$
f_q	$= 2 \frac{ B_{Aq} }{ B_f }, \text{ eq. (E.3), Appendix V}$
$f_{c1}, f_{s1}, f_{c2}, \dots$	Harmonics of (f/f_0)
G	Shear modulus, lbs/inch ²
G	$= 1/2 (Q+iq)$
H	$= 1/2 (Q-iq)$
h	Airgap at pole for eccentric rotor, inch
h_{kj}	Airgap at the j'th stator tooth of the k'th pole, inch
I	Transverse area moment of inertia of shaft section, inch ⁴
I_A	Effective or ammeter value of armature current, amp
I_{pn}	Polar mass moment of inertia of rotor station mass, lbs-inch-sec ²
I_{Tn}	Transverse mass moment of inertia of rotor station mass, lbs-inch-sec ²
I_0, I_1, I_2	Defined by eqs. (E-20) to (E.22), Appendix V
i	$= \sqrt{-1}$, the imaginary unit
i_A	Armature current, amp.
i_f	Field coil current, amp
K	Combined stiffness of rotor bearings, lbs/inch
K_d	Distribution factor for armature winding
K_p	Pitch factor for armature winding
$K_{xx}, K_{xy}, K_{yx}, K_{yy}$	Spring coefficients of bearing, lbs/inch
L_A	Inductance of line circuit, Henries
L_f	Inductance of field coil circuit, Henries
L_p	Distance between pole planes in homopolar generator, inch

l	Length of generator stator, inch
l	Rotor span between bearings, inch
l_n	Length of shaft section n, inch
M_x, M_y	x and y-component of rotor bending moment to the left of a rotor mass station, lbs-inch
M'_x, M'_y	x and y-component of rotor bending moment to the right of a rotor mass station, lbs-inch
m	Mass of a rigid, symmetric rotor, $\text{lbs}\cdot\text{sec}^2/\text{inch}$
N_A	Number of turns of a armature winding
N_f	Number of turns of a field coil
n	Number of generator north poles (=number of south poles)
n_r	Number of rotor teeth in the heteropolar generator
n_s	One half the number of stator teeth per pole in the heteropolar generator
P	$-\frac{\mu n_s A_T}{2C}$, permeance of airgaps in heteropolar generator
p_f	Power factor = $\cos \psi$
\bar{Q}	$-1/2,130,000$, conversion factor to get magnetic force in lbs.
Q, q	Matrices of magnetic force gradients, see eqs. (J.9) and (J.10), Appendix IX
Q_o	Negative stiffness of the static magnetic forces, lbs/inch
Q'_o	Negative moment stiffness of the static magnetic moments, lbs-inch/radian
Q_1	Stiffness of timevarying magnetic force, lbs/inch
Q_{ref}	Reference value used to represent the magnetic force stiffness in the stability calculation, see eq. (L.1)
$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}$	Cosine components of the radial stiffness of the magnetic forces, lbs/inch

$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}$	Cosine components of the radial stiffness of the magnetic forces, lbs/radians
$Q_{ox}, Q_{oy}, Q_{px}, Q_{py}$	Cosine components of the angular stiffness of the magnetic forces, lbs-inch/inch
$Q_{ox}, Q_{oy}, Q_{px}, Q_{py}$	Cosine components of the angular stiffness of the magnetic forces, lbs-inch/radian
$q_{xx}, q_{xy}, q_{yx}, q_{yy}$	Sine components of the radial stiffness of the magnetic forces, lbs/inch
$q_{xx}, q_{xy}, q_{yx}, q_{yy}$	Sine components of the radial stiffness of the magnetic forces, lbs/radian
$q_{ox}, q_{oy}, q_{px}, q_{py}$	Sine components of the angular stiffness of the magnetic forces, lbs-inch/inch
$q_{ox}, q_{oy}, q_{px}, q_{py}$	Sine components of the angular stiffness of the magnetic forces, lbs-inch/radian
R_A	Resistance of the line circuit, ohms
R_f	Resistance of the field coil circuit, ohms
R	Reluctance
R_N, R_S	Combined reluctance of the airgaps at the north and south poles
R_{Nk}, R_{Sk}	Reluctance of the airgap at the k'th north or south pole
R_{ak}, R_{bk}	Reluctances of the two airgaps at the k'th pole in the heteropolar generator
R_I	Reluctance of the flux path through the stator, see eq. (8)
R_R	Reluctance of the flux path through the rotor, see eq. (9)
R_1, R_2, R_3, R_4	Reluctances of airgaps 1,2,3 and 4 in the Two-Coil Lundell generator, see fig. 3
r	Radius , inch
S_k	Matrix used in the solution of the stability or response calculation, see eqs. (J.20) and (K.17)

s_0	Value of s_k for $k=0$, see eqs. (J.23) and (K.21)
s_{co}, s_{so}	$s_{co} + is_{so} = s_0$
T_x, T_y	x and y-component of magnetic moment in centerplane of alternator, lbs-inch
t	Time, seconds
v_x, v_y	x and y-component of rotor shear force to the left of a rotor mass station, lbs
v'_x, v'_y	x and y-component of rotor shear force to the right of a rotor mass station, lbs
\mathbf{x}	Vector representing the rotor amplitudes and slopes at the alternator centerplane, see eq. (H.69)
x_k	The k'th harmonic of X , see eqs. (J.7) and (K.6)
X_{ck}, X_{sk}	$X_k = X_{ck} + iX_{sk}$, see eqs. (J.8) and (K.7)
\mathbf{x}_1	Vector representing the amplitudes and slopes at the first rotor station, see eq. (H.63)
x, y	Rotor amplitudes , inch
x, y	Rotor amplitudes in the centerplane of the alternator, inch
x_o, y_o	x and y-component of the static rotor eccentricity in the centerplane of the alternator, inch
x'_o	Initial rotor eccentricity, inch
x_{ck}, x_{sk}	Cosine and sine component of the k'th harmonic of the rotor x-amplitude, inch
x_{ck}, y_{sk}	Cosine and sine component of the k'th harmonic of the rotor y-amplitude, inch
x_N, y_N	Rotor amplitudes in the plane of the north poles, inch
x_S, y_S	Rotor amplitudes in the plane of the south poles, inch
x_n, y_n	Rotor amplitudes at rotor mass station, n, inch

Y_A	Admittance of the line circuit, ohms ⁻¹
Y_F	Admittance of the field coil circuit, ohms ⁻¹
Z_A	Impedance of the line circuit, ohms
Z_F	Impedance of the field coil circuit, ohms
z	Coordinate along the rotor axis, inch
α	Angle between x-axis and direction of displacement
a	Cross-sectional factor for shear stress
β	$= [v^2 A / EI]^{1/4}$, inch ⁻¹
β_1, β_2	Defined by eq. (H.32), Appendix VIII, inch ⁻¹
γ_j	$\pm (4j-1) \frac{\pi}{2n_r}$
δ	$= [EI / 2aAG]^{1/2}$, inch
δ	Power angle for three-phase winding
ϵ	$= e/C$, eccentricity ratio
Θ	$= \frac{dx}{dz}$, slope of rotor in x-plane, inch/inch
Θ	Slope of rotor in x-plane at centerplane of alternator, inch/inch
Θ_0	Rotor misalignment angle in x-plane, inch/inch
Θ_n	Slope of rotor in x-plane at rotor station n, inch/inch
$X_{ii}, \lambda_{ii}, X_{i2}, \lambda_{i2}, \dots$	Elements of rotor impedance matrix E_k
X_k	$= K - Q_o - (kv)^2 n$
λ_k	$= kv B$
λ_1, λ_2	$= l_n \beta_1, = l_n \beta_2$
μ	Permeability of air
ν	Frequency, radians/sec
ρ	Mass density of shaft material, lbs-sec ² /in ⁴

ϕ	= $\frac{dy}{dz}$, slope of rotor in y-plane, inch/inch
ϕ	Slope of rotor in y-plane at centerplane of alternator, inch/inch
θ_0	Rotor misalignment angle in y-plane, inch/inch
θ_n	Slope of rotor in y-plane at rotor station n, inch/inch
Φ	Magnetic flux, lines
Φ_{Nk}, Φ_{Sk}	Flux at the k'th north pole or south pole, lines
Φ_{ak}, Φ_{bk}	Flux components at the k'th pole in heteropolar generator, lines.
ψ_1, ψ_3	Flux components for the two-coil Lundell generator, lines
Ω	Frequency of timevarying magnetic forces, radians/sec
ω	Angular speed of rotor, radians/sec
ω_c	Critical speed of symmetric, rigid rotor, radians/sec
$\omega_x, \omega_y, \omega_o, \omega_p$	Critical speeds of a rigid rotor, radians/sec

Subscripts

A	Armature and line circuit
a,b,c	Phase a,b,c of three phase winding, Appendix IV
f	Field coil circuit
c,s	Cosine and sine component (real and imaginary part)
j	Stator tooth number in heteropolar generator
k	Generator pole number
k	Frequency harmonic number
n	Rotor station number
N,S	North pole, south pole
x,y	In the x-direction or the y-direction
e, ϕ	In the e -direction or the ϕ -direction

Indices

j	Stator tooth number in heteropolar generator
k	Generator pole number
k	Frequency harmonic number
n	Rotor station number

INTRODUCTION

In the development of high speed electrical machinery for space power plants and compact power conversion machinery it has been found that the rotor may exhibit unsafe large amplitude vibration under certain operating conditions. This vibration is caused by the interaction between the rotor and the magnetic forces in the airgaps of the electrical machinery. It is further accentuated by the fact that the rotor bearings in this type of machinery employ low viscosity fluids or gas as a lubricant whereby the bearing stiffness and damping is smaller than for conventional bearings.

The application of alternators to this type of machinery is still in an early development phase and the experience with the vibration problem derives so far from machinery employing electrical motors. However, because of the close similarities between alternators and electrical motors it is to be expected that alternators may develop the same kind of vibration problems as previously found in motor applications. For this reason an analytical investigation of the problem as it could occur in alternators has been undertaken. It is the purpose of this volume to present an analysis of the magnetic forces in three brushless alternator types and to describe two computer programs which have been written to calculate the stability and vibratory response of an alternator rotor subjected to magnetic forces.

The three brushless generators investigated are: 1) the homopolar generator, 2) the heteropolar inductor generator, and 3) the two-coil Lundell generator. They are shown schematically in Figs. 1 to 3. Formulas are given from which the magnetic forces and moments can be calculated directly once the dimensions and operating conditions of the generator are known. Numerical examples are given to illustrate the practical use of the formulas.

Two computer programs have been written for calculating the dynamical performance of the alternator rotor with the imposed magnetic forces. The manuals for the programs, containing listings of the programs and the detailed instructions of how to use the programs and prepare the input data, are given in Appendices

XI and XII. The first program examines the stability of the alternator rotor to the generator magnetic forces, and the second program calculates the resulting amplitude response of the rotor when the axis of the rotor does not coincide with the magnetic axis of the generator stator. The programs are quite general and apply to any rotor or bearing configuration.

Because of the large number of parameters involved it is not possible to give any general results or design charts. However, from the few sample calculations performed and from certain simplified analyses, indications are that for most applications the rotor vibrations are small and the stability margin is good. On the other hand, if the operating conditions are such that the magnetic force frequency can excite a resonance of the rotor-bearing system, large rotor amplitudes may result and, furthermore, the stability margin may become unacceptable. A detailed calculation is required under these circumstances, and the methods presented in this volume provide the means for performing such calculations.

GENERAL DISCUSSION

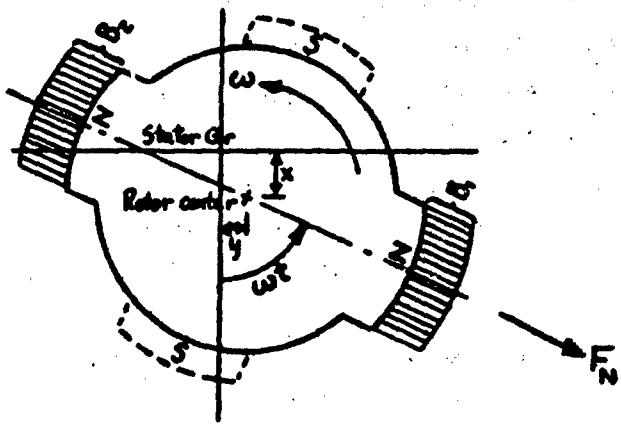
The problem of electromagnetically induced rotor vibrations was first encountered in a gas bearing supported, electrical motor driven compressor. Little was known about the causes of the vibrations and only by a trial test procedure were sufficient modifications made to the design that the unit performed satisfactorily. In two later applications, also motor driven compressors, serious vibrations were again encountered and, as in the first case, the problem could only be overcome by making trial modifications to the design until the vibrations were eliminated. In one of the cases, the problem was solved by changing the method of field excitation, proving that the problem definitely is caused by the electromagnetic forces.

When the need arose for incorporating alternators into this type of equipment it was natural that there were apprehensions about encountering the same vibration problems as already experienced with electrical motors. It has, therefore, been decided to undertake an investigation of the problem as it applies to alternators so that some design information is made available at an early state in the development of alternators for space power plants. Since the available experience with the problem is all based on electrical motors, the investigation is analytical and there are no test data to compare with.

The investigation falls naturally into two parts: a) a study of the magnetic forces in the alternator to establish methods and formulas from which the forces can be calculated, and b) the development of computational methods to determine the effect of the magnetic forces on the dynamics of the rotor.

The forces acting on the alternator rotor derive from the magnetic pull of the stator which is proportional to the square of the flux density in the airgap between the rotor and the stator. Hence, if the flux distribution is uniform around the rotor circumference the resultant magnetic force is zero. However, for the alternator to generate power it is necessary that the flux density varies around the circumference and, also, it must turn with the rotor. Even so, as long as the distribution is symmetric, no net forces will act on the rotor, but if the rotor is eccentric with respect to the stator, dissymmetries are introduced in the

flux distribution causing a resultant force on the rotor. To illustrate, consider a simple case of a four pole homopolar generator which is studied in more detail in Appendices I and V. The rotor has two north poles and two south poles which are not in the same plane (see also figure 1):



The flux density at the first north pole is B_1 , kilolines per sq. inch, and at the second north pole it is B_2 . If the area of a pole is A inch 2 , the net force acting on the rotor due to the north poles is:

$$F_N = \frac{A}{\eta C} (B_1^2 - B_2^2) \quad (1)$$

where the factor $\frac{1}{\eta C}$ is introduced to get F_N in lbs. when B_1 and B_2 are in kilolines per sq. inch and A is in sq. inch.

When the rotor is concentric within the stator, the radial airgap at the poles is C , inch, and the average flux density is B_0 . In that case, $B_1 = B_2 = B_0$ and the net force is zero (i.e. $F_N = 0$, see eq. (1)). However, when the rotor is eccentric such that its center has the coordinates x and y with respect to the center of the stator (see the figure above), the airgaps at the two poles are not the same. The flux density is inversely proportional to the airgap, and assuming the rotor eccentricity to be small compared to the radial gap C , the airgaps at the two poles can be expressed as:

$$\text{airgap at the first north pole: } h_1 = C \left[1 - \frac{x}{C} \cos(\omega t) - \frac{y}{C} \sin(\omega t) \right]$$

(2)

$$\text{airgap at the second north pole: } h_2 = C \left[1 + \frac{x}{C} \cos(\omega t) + \frac{y}{C} \sin(\omega t) \right]$$

ω is the angular speed of the rotor, radians/sec, such that (ωt) gives the angle at time t between the x -axis and the line through the poles. Since it is assumed that the ratios $\frac{x}{C}$ and $\frac{y}{C}$ are small compared to 1, the flux densities become:

$$B_1 = \frac{C}{h_1} B_o = B_o \left[1 + \frac{x}{C} \cos(\omega t) + \frac{y}{C} \sin(\omega t) \right]$$

(3)

$$B_2 = \frac{C}{h_2} B_o = B_o \left[1 - \frac{x}{C} \cos(\omega t) - \frac{y}{C} \sin(\omega t) \right]$$

from which:

$$B_1^2 = B_o^2 \left[1 + 2 \frac{x}{C} \cos(\omega t) + 2 \frac{y}{C} \sin(\omega t) \right]$$

$$B_2^2 = B_o^2 \left[1 - 2 \frac{x}{C} \cos(\omega t) - 2 \frac{y}{C} \sin(\omega t) \right]$$

The net force acting on the rotor is then determined from eq. (1) as:

$$F_N = 4 \frac{AB_o^2}{72C} [x \cos(\omega t) + y \sin(\omega t)] \quad (4)$$

This force follows the rotor as it turns. Its components in the fixed x - y -coordinate system are:

$$F_{Nx} = F_N \cos(\omega t) = 2 \frac{AB_o^2}{72C} [x(1 + \cos(2\omega t)) + y \sin(2\omega t)] \quad (5)$$

$$F_{Ny} = F_N \sin(\omega t) = 2 \frac{AB_o^2}{72C} [x \sin(2\omega t) + y(1 - \cos(2\omega t))]$$

It is seen that the force is directly proportional to the eccentricities x and y (this is, of course, only true when x and y are reasonably small compared to the radial airgap C). Hence, the magnetic force acts as a negative spring force (negative because the force acts in the same direction as the displacement; it does not oppose the displacement as a mechanical spring would do). It can further be observed that the force contains two parts, one part which for given eccentricities x and y is constant, and one part which varies periodically with a frequency of twice the rotor speed. The constant part of the force acts on the rotor simply as a negative, static spring, but the timevarying part can force the rotor to whirl and even induce instability as discussed

later. All generator types produce magnetic forces of the same general form as shown above although the timevarying part may be absent in some cases. Of the three generator types studies, the four pole homopolar generator and the heteropolar inductor generator under load have magnetic forces with timevarying components, whereas the homopolar generator with more than four poles and the two-coil Lundell generator only have static force components.

To complete the above example of the magnetic forces in the four pole homopolar generator, where only the forces acting on the north poles have been considered so far, the forces acting on the south poles, F_{sx} and F_{sy} , can be obtained simply by observing that the south poles lag the north poles by 90 degrees. Hence, by replacing (ωt) by ($\omega t - 90^\circ$) in eqs. (5), the forces on the south poles become:

$$\begin{aligned} F_{sx} &= 2 \frac{AB^2}{72C} [x(1-\cos(2\omega t)) - y \sin(2\omega t)] \\ F_{sy} &= 2 \frac{AB^2}{72C} [-x \sin(2\omega t) + y(1+\cos(2\omega t))] \end{aligned} \quad (6)$$

Thus, if the north poles and the south poles are in the same plane, the net forces acting on the rotor, namely $F_x = (F_{nx} + F_{sx})$ and $F_y = (F_{ny} + F_{sy})$, are independent of time. Only, when the pole planes do not coincide, is there a possibility of a timevarying force or a timevarying moment. For details, see Appendix I.

This simple analysis illustrates the general character of the more detailed analysis employed in Appendices I to VI to derive the formulas for calculating the magnetic forces. The analyses are concerned with the fundamental harmonic of the forces and do not consider the contributions from such factors as higher harmonics in the flux wave or higher harmonics in the flux density distribution caused by edge effects, slotting or pole shape. It will normally be found that such factors have negligible influence on the net forces although they can cause appreciable local forces. They are basically unaffected by rotor eccentricity and, therefore, cancel out when the net force is obtained. The effect of generator load, on the other hand, cannot be ignored and is included in the analysis. When the alternator operates under load, the armature windings produce a magnetomotive force, commonly known as the armature reaction, which opposes the flux direction of the main field. Thus, the general effect of the armature reaction is to reduce the magnetic forces and, at the same time, it may also introduce new timevarying force components

which have the frequency of the line current or harmonics thereof.

The analysis ignores saturation effects in the iron and assumes that all of the reluctance in the magnetic flux path occurs in the airgaps at the poles. In practice this assumption is not valid where the iron operates close to saturation. The effect of saturation is to reduce the magnetic force values obtained on the basis of unsaturated iron. To illustrate, return to the homopolar generator analyzed above. As shown in Figure 1, the field coil is between the two pole planes. The flux starts in the rotor at the north poles, passes the airgaps of the north poles, goes through the stator iron and returns to the rotor via the airgaps of the south poles. The combined reluctance of the two airgaps of the north poles is (two reluctances in parallel):

$$R_N = \frac{C}{2\mu A} \quad (7)$$

where C is the radial airgap, A is the pole area and μ is the permeability of air. The reluctance R_S of the airgaps at the south poles is the same, i.e. $R_S = R_N$. The reluctance R_I of the flux path through the stator iron depends on how the flux is distributed over the cross-sections of the path. Symbolically the reluctance may be written:

$$R_I = \int_0^{l_I} \frac{dz}{\mu_I A_I} \approx \frac{l_I}{(\mu_I A_I)_{\text{average}}} \quad (8)$$

where l_I represents the total length of the flux path, z is the coordinate along the path, A_I is the cross-sectional flux area which depends on z and μ_I is the permeability of the stator iron. μ_I is a function of the local flux density and, thus, is a function of z (it actually also varies over the cross-sectional area). The calculation of R_I is rather complicated since the permeability is a non-linear function of the flux density, thereby making it necessary to compute the detailed flux distribution in order to find the effective overall reluctance R_I of the flux path.

The reluctance R_R of the flux path through the rotor can be represented in a similar way:

$$R_R \approx \frac{l_R}{(\mu_R A_R)_{\text{average}}} \quad (9)$$

where the meaning of the symbols is the same as above. The total reluctance of the flux path is the sum of the four reluctances: $R_N + R_S + R_I + R_R$. Thus, if the field coil produces a magnetomotive force \mathcal{F}_f , the drop in mmf, \mathcal{F}_N , across the airgaps at the north poles become:

$$\mathcal{F}_N = \frac{R_N}{R_N + R_S + R_I + R_R} \mathcal{F}_f \quad (10)$$

where $R_S = R_N$. If saturation effects are ignored, then $R_I = R_R = 0$ and $\mathcal{F}_N = \frac{1}{2} \mathcal{F}_f$. The actual drop in mmf is smaller, causing a proportional reduction in flux density and, therefore, a parallel reduction in the magnetic forces. In this way the effect of saturation can be included by multiplying the force values obtained on the basis of unsaturated iron by the factor: $[R_N / (2R_N + R_I + R_R)]^2$. It should be emphasized, however, that this adjustment is only necessary if the flux density has been calculated on the basis of unsaturated iron. If instead the actual flux density, B_0 , in the airgaps at the poles is known and used directly in the formulas given in the following sections, the effect of saturation is already included (because the effect is included in B_0).

Having determined the magnetic forces, their effect on the rotor can be studied. It should first be noted that that part of the forces which does not vary with time, has only a "passive" effect. It can be represented simply by negative springs in parallel with the stiffnesses of the rotor bearings and the stiffness of the rotor shaft. It is obvious that if this negative stiffness is large enough to offset the combined rotor-bearing stiffness, then the rotor will be statically unstable or, in other words, the magnetic forces are so large that the rotor is simply pulled up against the stator. This case is of academic interest only or, at least, it is readily checked and does not require any special investigation. Hence, assuming that the system is statically stable, the timeindependent components of the magnetic forces can be considered an integral part of the rotor-bearing system in which they are included simply as another rotor bearing, although with a negative stiffness. In the analysis this negative stiffness is called Q_0 ,

lbs/inch, and in addition allowance is made for a similar negative moment stiffness Q'_e , lbs-inch/radian. They must be specified in the input to the rotor computer programs.

Turning next to the timevarying components of the magnetic forces, they can influence the rotor in two ways: they may induce instability and, furthermore, if there is any built-in eccentricity between the rotor and the stator, they can force the rotor to whirl. Using again the previous example of the 4 pole homopolar generator, let the rotor displacements measured from the center of the alternator stator be x_N and y_N in the plane of the north poles, and x_s and y_s in the plane of the south poles. Then the timevarying components of the magnetic forces can be written (from eqs. (5) and (6)):

$$\begin{aligned}(F_{Nx})_{\text{timevarying}} &= \frac{AB_o^2}{36C} [x_N \cos(2\omega t) + y_N \sin(2\omega t)] \\ (F_{Ny})_{\text{timevarying}} &= \frac{AB_o^2}{36C} [x_N \sin(2\omega t) - y_N \cos(2\omega t)] \\ (F_{sx})_{\text{timevarying}} &= -\frac{AB_o^2}{36C} [x_s \cos(2\omega t) + y_s \sin(2\omega t)] \\ (F_{sy})_{\text{timevarying}} &= -\frac{AB_o^2}{36C} [x_s \sin(2\omega t) - y_s \cos(2\omega t)]\end{aligned}\tag{11}$$

Let the distance between the two pole planes be L_p , inch. Then the slopes of the rotor are:

$$\begin{aligned}\theta &= \frac{1}{L_p} (x_N - x_s) \\ \phi &= \frac{1}{L_p} (y_N - y_s)\end{aligned}\tag{12}$$

The rotor displacements at the center plane of the alternator, midway between the two pole planes, are:

$$\begin{aligned}x &= \frac{1}{2} (x_N + x_s) \\ y &= \frac{1}{2} (y_N + y_s)\end{aligned}\tag{13}$$

The forces F_x and F_y , and the moments T_x and T_y acting on the rotor at the centerplane of the alternator are:

$$\begin{aligned} F_x &= F_{Nx} + F_{Sx} \\ F_y &= F_{Ny} + F_{Sy} \\ T_x &= \frac{1}{2} L_p (F_{Nx} - F_{Sx}) \\ T_y &= \frac{1}{2} L_p (F_{Ny} - F_{Sy}) \end{aligned} \quad (14)$$

Substituting eqs. (11) into eqs (14) and making use of eqs. (12) and (13) yields:

$$\begin{aligned} F_x &= \frac{AB^2}{36C} L_p [\theta \cdot \cos(2\omega t) + \phi \sin(2\omega t)] \\ F_y &= \frac{AB^2}{36C} L_p [\theta \cdot \sin(2\omega t) - \phi \cos(2\omega t)] \\ T_x &= \frac{AB^2}{36C} L_p [x \cdot \cos(2\omega t) + y \sin(2\omega t)] \\ T_y &= \frac{AB^2}{36C} L_p [x \sin(2\omega t) - y \cos(2\omega t)] \end{aligned} \quad (15)$$

Now, assume that the rotor starts to whirl in a closed orbit such that x, y, θ , and ϕ represent harmonic oscillations. Then the magnetic forces perform work on the rotor and, if this work is integrated over one cycle to determine the net energy, it will be found, that if the rotor motion is periodic with a fundamental frequency of either ω or 2ω , the possibility exists of the energy input to the rotor being positive. In other words, energy can be transferred from the magnetic field to the rotor motion. It obviously depends on the phase relationships between the magnetic forces and the rotor motion if the energy transfer to the rotor will be positive or negative, but if the energy is positive, the rotor motion will actually grow and the rotor is unstable. If the energy is zero, the rotor motion will persist indefinitely and the system is on the threshold of instability. The phase relationship between the magnetic forces and the rotor motion is governed by the stiffness, damping and inertia properties of the rotor-bearing system and for an actual rotor it is necessary to perform the detailed calculations on a computer. The computer program for such a calculation is described in details in Appendix XI. Although the program does not actually check the rotor stability by means of the outlined energy method, the employed method is equivalent and the basic analysis is described in Appendix IX. The program calculates the threshold of instability as the zero-point of either of two determinants in complete analogy to the above development where the rotor is on the threshold of instability when

the energy input is zero which may happen when the fundamental frequency is either ω or 2ω (in fact, the two determinants corresponds to rotor motions with these two fundamental frequencies). In searching for the zero points of the determinants (i.e. the threshold of instability); the rotor speed is kept fixed and the magnetic forces are varied over a specified range. Once the threshold has been found, it is immediately checked if the actual magnetic forces puts the rotor in a stable or an unstable zone of operation.

This form of instability is normally classified as a Mathieu type of instability (references 1 and 2). However, the classical Mathieu equation represents, in this context, a rigid, symmetrical rotor with only one critical speed, with only one amplitude direction and with no damping in the system. A stability map for such a system, although with damping included, is shown in figure 5. It will be discussed in detail in the chapter entitled "The Concept of Stability and Response of a Rotor with Magnetic Forces". Most rotors, however, cannot be represented in this simplified fashion. A typical rotor is not entirely symmetric and of greater importance, it has many critical speeds. Furthermore, the Mathieu equation allows only for one form of the magnetic forces which, as an example, cannot be made to represent the forces in a four pole homopolar generator. The developed computer program is far more general and can treat any arbitrary rotor with all its resonances, and also makes it possible to specify magnetic forces of any form desired.

In writing the program it has been considered to admit more than one frequency in the magnetic forces (in the language of the literature: to go from a Mathieu equation to a Hill equation). However, the study of the three selected generator types does not indicate that higher harmonics of the fundamental frequency are important. Hence, only the force components of the fundamental harmonic are treated by the computer program. Furthermore, to include higher harmonics would drastically increase the computer time. From the point-of-view of the analysis or writing the computer program, it is just as easy to treat any number of frequencies than just one but it is felt, at this stage, that it would be unjustified because of the computer time.

It should be emphasized that it is not a simple routine matter to perform a stability calculation. It requires some pre-knowledge of the characteristics of the rotor-bearings system (notably where the critical speeds are) in order to interpret the results of the calculations correctly, and it is necessary to perform calculations not only at the operating speed but over a sufficient range of speeds that a stability map can be established. These problems are discussed at length in the chapter entitled: "Discussion on Performing Stability Calculations."

The rotor response calculation is more readily performed than the stability calculation. Let eqs. (15) be representative of the timevarying magnetic forces and assume that there is a built-in eccentricity between the rotor centerline and the stator axis such that, without the rotor whirling, the center of the rotor has the coordinates x_0 and y_0 with respect to the center of the stator, measured in the centerplane of the alternator. Furthermore, the axis of the rotor is misaligned with respect to the stator axis by the angles θ_0 and ϕ_0 . Then, as seen from eqs. (15), forces and moments will act on the rotor:

$$\begin{aligned}(F_x)_0 &= \frac{AB^2}{36C} L_p [x_0 \cos(2\omega t) + y_0 \sin(2\omega t)] \\ (F_y)_0 &= \frac{AB^2}{36C} L_p [y_0 \sin(2\omega t) - x_0 \cos(2\omega t)] \\ (T_x)_0 &= \frac{AB^2}{36C} L_p [x_0 \cos(2\omega t) + y_0 \sin(2\omega t)] \\ (T_y)_0 &= \frac{AB^2}{36C} L_p [x_0 \sin(2\omega t) - y_0 \cos(2\omega t)]\end{aligned}\quad (16)$$

These forces and moments will obviously force the rotor to whirl. Since the forces have a frequency of 2ω , radians/sec, it is to be expected that the rotor will whirl with the same frequency, i.e. the rotor vibrations will be at twice per revolution instead of the synchronous vibration encountered when the rotor has a mechanical unbalance. In general it will be found that this will be the predominant frequency of the vibration. However, as shown by eq. (15), the magnetic forces depend on the rotor amplitude. Eqs. (16) only represents that part of the forces which is induced by the built-in eccentricity. The remaining part of the magnetic forces, namely the difference between eqs. (15) and (16), will interact with the induced forces via the rotor-bearing system and will cause the rotor to respond not only with a frequency of 2ω , but also with the frequencies $4\omega, 6\omega, 8\omega$ and so on. This calculation is performed by means of the computer program described in details in Appendix XII. The basic analysis is contained in Appendix X. As in

the stability computer program, the rotor may be any arbitrary flexible rotor with several bearings, and the form of the magnetic forces is quite general. To calculate the response, the built-in eccentricities between the rotor and the stator must, of course, be specified. A more detailed discussion of this type of calculation is given in the chapter: "Discussion on Performing Response Calculations."

In summary it can be said that the analyses, the formulas for calculating the magnetic forces in the generator and the two rotor dynamic computer programs are quite general, and together they provide adequate means for a comprehensive check of the performance of a proposed alternator rotor-bearing system design. It should be noted, however, that whereas the presented analyses and the computational methods are believed to be sound, there are no test data or actual measurements available against which the theoretical predictions can be checked and compared. Furthermore, in those applications where the magnetic force gradients are small compared to the combined rotor-bearing stiffness (say, less than 30 percent), the two rotor computer programs are unnecessarily "sophisticated" and unjustifiably complex. They give correct results but in far more detail than required for design purposes. On the other hand it is felt, that as long as no real practical experience is available to serve as a guide, it is safer to use calculation methods which are generally applicable although for any particular application much of the generated information may prove to be of limited practical significance. At this early time in the development of designing alternators for space power plants it is not possible to predict with any accuracy what future requirements may demand and viewed in that context the presented methods should be able to serve their purpose.

THE MAGNETIC FORCES OF THREE BRUSHLESS GENERATOR TYPES

Three brushless generator types are investigated: the homopolar generator, the heteropolar inductor generator and the two-coil Lundell generator. They are shown schematically in Figures 1 to 3. The magnetic forces produced by these generators have been derived for the generators operating without and with load and the analyses are given in detail in Appendices I to III. Since manufactured generators differ widely in construction (notably in the way they are wound), even if they are of the same type, the analyses are kept general, not specific. The objective of the analyses is to derive simple formulas from which the magnetic forces can be calculated with sufficient accuracy for engineering purposes. Only the fundamental harmonic of the forces are considered and such factors as higher harmonics in the flux wave or stator and rotor slotting are disregarded. The most serious assumption is that saturation effects are ignored. However, saturation will reduce the magnitude of the forces and the developed formulas will, therefore, give too large forces. Thus, the rotor calculations will be conservative and actually have a built-in safety factor. It should be noted, on the other hand, that in an actual generator the eccentricity between the stator and the rotor is not known with too high a degree of accuracy, so that a safety factor is required anyway.

The formulas for calculating the magnetic forces are set up to conform with the required input format to the rotor stability and the rotor response computer programs. Hence, the calculated numerical values can be used directly as input to either of the two computer programs. To explain the input format, let x and y be the amplitudes of the rotor in the centerplane of the generator, and let Θ and Φ be the corresponding slopes of the rotor (i.e. $\Theta = \frac{dx}{dz}$ and $\Phi = \frac{dy}{dz}$, taken at the centerplane of the generator, where z is the coordinate along the rotor axis). The magnetic forces have the two force components: F_x and F_y , and the two moment components: T_x and T_y . The forces are assumed to be proportional to the amplitudes and slopes of the rotor:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} Q_x \\ Q_y \\ Q_\Theta \\ Q_\Phi \end{Bmatrix} - \left[\begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\Theta} & Q_{x\Phi} \\ Q_{yx} & Q_{yy} & Q_{y\Theta} & Q_{y\Phi} \\ Q_{x\Theta} & Q_{y\Theta} & Q_{\Theta\Theta} & Q_{\Theta\Phi} \\ Q_{x\Phi} & Q_{y\Phi} & Q_{\Theta\Phi} & Q_{\Phi\Phi} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\Theta} & q_{x\Phi} \\ q_{yx} & q_{yy} & q_{y\Theta} & q_{y\Phi} \\ q_{x\Theta} & q_{y\Theta} & q_{\Theta\Theta} & q_{\Theta\Phi} \\ q_{x\Phi} & q_{y\Phi} & q_{\Theta\Phi} & q_{\Phi\Phi} \end{Bmatrix} \sin(\Omega t) \right] \begin{Bmatrix} x \\ y \\ \Theta \\ \Phi \end{Bmatrix} \quad (17)$$

Here, F_x and F_y are the magnetic forces in lbs, T_x and T_y are the magnetic moments in lbs-inch, Ω is the frequency of the magnetic forces in radians/sec., t is time in seconds, and the Q's and q's are the gradients of the magnetic forces and moments where the first index specifies the force direction and the last index gives the amplitude direction. The two computer programs require that the values of these gradients are given in the computer input. In the following it will be shown how the gradients are obtained for the generator types under study.

The 4 Pole Homopolar Generator - The magnetic forces in the homopolar generator are analyzed in Appendices I and V. There it is shown that only for the four pole generator are the magnetic forces timevarying. Thus, the rotor stability program and the rotor response program only apply to this case. For a different number of poles, there is no response or stability problem.

As shown in Figure 1, the north poles and the south poles are in separate planes. Let the distance between the two planes be L_p , inch. Then the magnetic forces and moments for the generator operating with no load become (see Eq. (A.46),

Appendix I.):

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \frac{AB^2}{36C} \begin{Bmatrix} 2x \\ 2y \\ \frac{1}{2}L_p^2 \theta \\ \frac{1}{2}L_p^2 \phi \end{Bmatrix} - L_p \begin{Bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Bmatrix} \cos(2\omega t) - \begin{Bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{Bmatrix} \sin(2\omega t) \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix} \quad (18)$$

where ω is the angular speed of the rotor in radians/sec. Hence, in terms of eq. (17):

$$Q_o = 4 \frac{AB^2}{72C} \frac{\text{lbs}}{\text{inch}} \quad (19)$$

$$Q'_o = \frac{AB^2}{72C} L_p^2 \frac{\text{lbs-inch}}{\text{radian}} \quad (20)$$

$$-Q_{xe} = Q_{yy} = -Q_{ex} = Q_{xy} = q_{xe} = q_{yy} = q_{ex} = q_{xy} = 2 \frac{AB^2}{72C} L_p \quad \text{lbs} \quad (21)$$

(all other values of Q and q are zero)

The nomenclature is:

$$A = \text{area of one pole, inch}^2$$

C = radial air gap at the poles, inch

L_p = distance between planes of north poles and south poles, inch.

B_c = Average flux density in the air gaps at the poles due to the field coils, kilolines/inch²

To take an example, let the average flux density be 50 kilolines/inch².

(This is a rather average value for generators for space power plants and similar applications). Furthermore, let the radial air gap be 0.040 inch,

the length between pole planes: $L_p = 4.2$ inch and the pole area: $A = 5.76$ inch².

Then:

$$\frac{AB_A^2}{72C} = \frac{5.76 \cdot (50)^2}{72 \cdot 0.040} = 5,000 \frac{\text{lbs}}{\text{inch}}$$

Hence:

$$Q_0 = 20,000 \frac{\text{lbs}}{\text{inch}}$$

$$Q'_0 = 88,200 \frac{\text{lbs-inch}}{\text{radian}}$$

$$-Q_{x0} = Q_{y0} = q_{x0} = q_{y0} = 42,000 \frac{\text{lbs}}{\text{radian}}$$

$$-Q_{ex} = Q_{ey} = q_{ex} = q_{ey} = 42,000 \frac{\text{lbs-inch}}{\text{inch}}$$

The ratio between the magnetic force frequency and the speed of the rotor is:

$$\frac{\Omega}{\omega} = 2$$

In this way all the magnetic force input data required for the two computer programs have been obtained.

When the generator is loaded, all the above values are reduced by being multiplied by the factor $(1-f_d)^2$ where:

$$f_d = \frac{12\sqrt{2}}{\pi} \frac{R_f}{N_f E_f} K_d K_p N_A I_A \sin(\gamma + \delta) \quad (23)$$

where:

R_f = resistance of the field coil, ohms.

N_f = number of turns of the field coil

E_f = the d.c. voltage impressed in the field coils, volts

N_A = number of turns of one armature winding.

I_A = the ammeter value of the current, amps

K_d = distribution factor (≈ 0.96 to 1)

K_p = pitch factor (≈ 0.96 to 1)

ψ = power factor angle

δ = power angle.

Thus, f_4 gives the ratio between the de-magnetizing component of the armature reaction and half of the mmf of the field coil. A detailed discussion is given in Appendices IV and V. To calculate f_4 it is necessary to obtain the necessary data from the generator manufacturer.

The Heteropolar Inductor Generator under Load. The magnetic forces of the heteropolar inductor generator under load are analyzed in Appendix VI. Only when the generator operates under load are the magnetic forces time-dependent and, thus, only in this case can a rotor stability and a rotor response calculation be performed.

The heteropolar inductor generator produces magnetic forces only and no moments.

The forces are derived in Appendix VI as:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{2nn_s A_r B^2}{72C} \left[\left(1 + \frac{1}{2} f_{c1}^2 + \frac{1}{2} f_{s1}^2 \right) \begin{Bmatrix} x \\ y \end{Bmatrix} - \left[2f_{c1} \begin{Bmatrix} -1 & 0 \\ 0 & -1 \end{Bmatrix} \cos(\eta_s \omega t) - 2f_{s1} \begin{Bmatrix} -1 & 0 \\ 0 & -1 \end{Bmatrix} \sin(\eta_s \omega t) \right] \begin{Bmatrix} x \\ y \end{Bmatrix} \right] \quad (24)$$

Hence, in terms of eq. (17):

$$Q_o = \frac{2nn_s A_r B^2}{72C} \left[1 + \frac{1}{2} (f_{c1}^2 + f_{s1}^2) \right] \quad \frac{\text{lbs}}{\text{inch}} \quad (25)$$

$$Q'_o = 0 \quad (26)$$

$$Q_{xx} = Q_{yy} = - \frac{2nn_s A_r B^2}{72C} 2f_{c1} \quad \frac{\text{lbs}}{\text{inch}} \quad (27)$$

$$q_{xx} = q_{yy} = - \frac{2nn_s A_r B^2}{72C} 2f_{s1} \quad \frac{\text{lbs}}{\text{inch}} \quad (28)$$

All other gradients are zero. The ratio between the magnetic force frequency Ω and the rotor speed is:

$$\frac{\Omega}{\omega} = n_r$$

(29)

The nomenclature is:

- $2n$ = total number of poles (n north poles and n south poles)
- n_s = one half the number of stator teeth per pole
- n_r = total number of rotor teeth
- A_T = area of one stator tooth, inch
- C = radial air gap at the poles, inch
- B_o = average flux density in the air gaps at the poles due to the field coils, kilolines/inch²
- f_{c1}, f_{s1} = fundamental components of the armature reaction, dimensionless.

The method for evaluating f_{c1} and f_{s1} is given in Appendix VI. Here it is found, restricting the analysis to the first harmonic, that:

$$f_{c1} + i f_{s1} = \frac{i \nu Y_A L_c}{1 + i \nu Y_f L_f} \quad (30)$$

where:

$$i = \sqrt{-1}$$

ν = $n_r \omega$, the frequency of the magnetic forces, radians/sec

L_c = sum of the self-inductances of the $4n$ armature coils, Henries

L_f = the self-inductance of one field coil, Henries

$Y_f = \frac{1}{R_f + i \nu L_f}$, the admittance of a field coil, ohms⁻¹

R_f = resistance of a field coil, ohms

$Y_A = \frac{1}{R_A + i \nu L_A}$, the admittance of the power circuit, ohms⁻¹

R_A = resistance of the power circuit, ohms

L_A = inductance of the power circuit, Henries

Let P be the permeance of the airgaps of one half the stator teeth of a pole.

Then:

$$P = \frac{\mu n_r A_T}{2C} \quad (31)$$

where μ is the permeability of air. Then:

$$L_c = 4\pi \mu N_A^2 \quad (32)$$

$$L_f = \frac{1}{2} \mu N_f^2 \quad (33)$$

where:

N_A = number of turns of one armature coil (there are $4n$ coils)

N_f = number of turns of one field coil

Returning to eq. (30), it can be written:

$$f_{c1} + if_{s1} = \frac{\frac{i\sqrt{L_c}}{R_A + i\sqrt{L_A}}}{1 + \frac{i\sqrt{L_c}}{R_f + i\sqrt{L_f}}} = \frac{\frac{\sqrt{L_c}/\sqrt{L_A}}{1 - i\frac{R_A}{\sqrt{L_A}}/\sqrt{L_A}}}{1 + \frac{1}{1 - i\frac{R_f}{\sqrt{L_f}}/\sqrt{L_f}}} \quad (34)$$

Consider the numerator. When the line current lags the line voltage by a phase angle ψ , then the power factor, pf, is defined as:

$$pf = \cos \psi \quad (35)$$

As shown in Appendix IV eq. (D.15):

$$pf = \frac{R_A}{\sqrt{R_A^2 + (\sqrt{L_A})^2}} \quad (36)$$

for which:

$$\frac{R_A}{\sqrt{L_A}} = \frac{pf}{\sqrt{1 - (pf)^2}} \quad (37)$$

If it assumed, furthermore, that the field coil circuit is predominantly inductive (i.e. $R_f/\sqrt{L_f} \approx 0$), then eq. (34) becomes:

$$f_{c1} + if_{s1} \approx \frac{1}{2} \frac{L_c}{L_A} \frac{\sqrt{1 - (pf)^2}}{\sqrt{1 - (pf)^2} - i(pf)} \quad (38)$$

from which:

$$f_{c_1} \approx \frac{1}{2} \frac{L_c}{L_A} [1 - (pf)^2] \quad (39)$$

$$f_{s_1} \approx \frac{1}{2} \frac{L_c}{L_A} \cdot (pf) \cdot \sqrt{1 - (pf)^2} \quad (40)$$

These equations must be considered to be approximate only, but they probably yield sufficient accurate results for the purpose of the rotor calculations.

To illustrate the use of the expressions, consider a numerical example. Let the generator configuration be as depicted in Fig. 2. Hence, there are 4 poles with four stator teeth per pole, and the rotor has 20 teeth:

$$n = 2$$

$$n_s = 2$$

$$n_r = 20$$

The rotor is 6 inches long and the area of one stator tooth is 1.25 inch². The radial airgap is 0.005 inch:

$$A_T = 1.25 \text{ inch}^2$$

$$C = 0.005 \text{ inch}$$

Assume an average flux density of:

$$B_o = 50 \text{ kilolines/inch}^2$$

With these numbers:

$$\frac{2nn_s A_T B_o^2}{72C} = \frac{2 \cdot 2 \cdot 2 \cdot 1.25 \cdot (50)^2}{72 \cdot 0.005} = 69,440 \frac{\text{lbs}}{\text{inch}}$$

Assume the power factor to be 0.8 and let the ratio between the inductance of the armature coils and the power circuit be 1:

$$pf = 0.8$$

$$L_c/L_A = 1$$

Then, from eqs. (39) and (40):

$$f_{c1} = 0.18$$

$$f_{s1} = 0.24$$

Then, eqs. (25) to (29) yield:

$$Q_0 = 75,690 \frac{\text{lbs}}{\text{inch}}$$

$$Q'_0 = 0$$

$$Q_{xx} = Q_{yy} = 25,000 \frac{\text{lbs}}{\text{inch}}$$

$$q_{xx} = q_{yy} = 33,300 \frac{\text{lbs}}{\text{inch}}$$

$$\frac{\Omega}{\omega} = 20$$

(41)

These values can be used directly as input to the rotor response and the rotor stability programs.

The Two-Coil Lundell Generator. The magnetic forces for this generator are analyzed in Appendix III. It is shown that, in general, there are no timevarying magnetic forces in the two-coil Lundell generator or, if there are, they will be small since they are caused primarily by differences in pole areas. Hence, this generator is of little interest for purposes of calculating rotor stability and rotor response. The two-coil Lundell generator has the same magnetic force characteristics as the homopolar generator except that the north poles and the south poles are in the same plane. It is this factor which is responsible for eliminating the time-averaging magnetic forces.

THE CONCEPT OF STABILITY AND RESPONSE OF A ROTOR WITH MAGNETIC FORCES

In the preceding chapter, a brief discussion has been given on how the magnetic forces can cause the rotor to whirl and also induce instability. A more specific discussion is, however, necessary in order to describe the basic features of the two rotor computer programs. For this purpose, consider a simplified rotor model where the rotor is rigid and symmetric. The rotor mass is m , the total bearing stiffness is K and the total bearing damping is B . If the rotor amplitude is x the magnetic forces are taken as: $(Q_0 - Q_1 \cos(\Omega t))x$ where Ω is the frequency of the magnetic forces, Q_0 and Q_1 are the gradients of the magnetic force, and t is time. The equation of motion is:

$$m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + [K - Q_0 + Q_1 \cos(\Omega t)]x = 0 \quad (42)$$

This equation is the damped Mathieu equation. To check its stability, expand x in a Fourier series (references 1 and 2):

$$x = \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] \quad (43)$$

where:

$$\psi = \frac{1}{2}\Omega t = vt \quad (44)$$

$$v = \frac{1}{2}\Omega \quad (45)$$

Substitute eq. (43) into eq. (42):

$$\sum_{k=0}^{\infty} \left\{ [(K - Q_0 - (kv)^2 m)x_{ck} - kvBx_{sk}] \cos(k\psi) - [(K - Q_0 - (kv)^2 m)x_{sk} + kvBx_{ck}] \sin(k\psi) \right\} \\ + Q_1 \cos(2\psi) \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] = 0 \quad (46)$$

With the trigonometric identities:

$$\begin{aligned} \cos(2\psi) \cos(k\psi) &= \frac{1}{2} [\cos((k+2)\psi) + \cos((k-2)\psi)] \\ \cos(2\psi) \sin(k\psi) &= \frac{1}{2} [\sin((k+2)\psi) + \sin((k-2)\psi)] \end{aligned} \quad (47)$$

and collecting terms in $\cos(k\psi)$ and $\sin(k\psi)$, eq. (46) gives rise to two infinite sets of simultaneous equations. Define:

$$x_k = K - Q_0 - (k\nu)^2 m = K - Q_0 - \left(\frac{k\Omega}{2}\right)^2 m \quad (48)$$

$$\lambda_k = k\nu B = \frac{k\Omega}{2} B \quad (49)$$

whereby the two sets of equations can be written:

$$\begin{Bmatrix} x_0 & tQ_1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ Q_1 & x_2 & -\lambda_2 & tQ_1 & 0 & 0 & 0 & 0 & \dots \\ 0 & \lambda_2 & x_2 & 0 & tQ_1 & 0 & 0 & 0 & \dots \\ 0 & tQ_1 & 0 & x_4 & -\lambda_4 & tQ_1 & 0 & 0 & \dots \\ 0 & 0 & tQ_1 & \lambda_4 & x_4 & 0 & \frac{1}{2}Q_1 & 0 & \dots \\ 0 & 0 & 0 & \frac{1}{2}Q_1 & 0 & x_6 & -\lambda_6 & tQ_1 & 0 & \dots \\ 0 & 0 & 0 & 0 & \frac{1}{2}Q_1 & \lambda_6 & x_6 & 0 & \frac{1}{2}Q_1 & \dots \\ \vdots & \vdots \\ \vdots & \vdots \end{Bmatrix} \begin{Bmatrix} x_{c0} \\ x_{c2} \\ x_{s2} \\ x_{c4} \\ x_{s4} \\ x_{c6} \\ x_{s6} \end{Bmatrix} = 0 \quad (50)$$

$k=0,2,4,\dots$

$$\begin{Bmatrix} (x_1 + \frac{1}{2}Q_1) & -\lambda_1 & \frac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & \dots \\ \lambda_1 & (x_1 - \frac{1}{2}Q_1) & 0 & tQ_1 & 0 & 0 & 0 & 0 & \dots \\ \frac{1}{2}Q_1 & 0 & x_3 & -\lambda_3 & tQ_1 & 0 & 0 & 0 & \dots \\ 0 & \frac{1}{2}Q_1 & \lambda_3 & x_3 & 0 & tQ_1 & 0 & 0 & \dots \\ 0 & 0 & \frac{1}{2}Q_1 & 0 & x_5 & -\lambda_5 & tQ_1 & 0 & \dots \\ 0 & 0 & 0 & \frac{1}{2}Q_1 & \lambda_5 & x_5 & 0 & \frac{1}{2}Q_1 & \dots \\ \vdots & \vdots \\ \vdots & \vdots \end{Bmatrix} \begin{Bmatrix} x_{c1} \\ x_{s1} \\ x_{c3} \\ x_{s3} \\ x_{c5} \\ x_{s5} \end{Bmatrix} = 0 \quad (51)$$

$k=1,3,5,\dots$

To obtain a solution, note that for $k \geq 3$ the equations can be written:

$$\underline{k \geq 3} \quad \frac{1}{2} \begin{Bmatrix} Q_1 & 0 \\ 0 & Q_1 \end{Bmatrix} \left[\begin{Bmatrix} x_{c,k-2} \\ x_{s,k-2} \end{Bmatrix} + \begin{Bmatrix} x_{c,k+2} \\ x_{s,k+2} \end{Bmatrix} \right] + \begin{Bmatrix} x_k & -\lambda_k \\ \lambda_k & x_k \end{Bmatrix} \begin{Bmatrix} x_{ck} \\ x_{sk} \end{Bmatrix} = 0 \quad (52)$$

Set:

$$\begin{Bmatrix} x_{ck} \\ x_{sk} \end{Bmatrix} = \begin{Bmatrix} \alpha_{k-2} & -\beta_{k-2} \\ \beta_{k-2} & \alpha_{k-2} \end{Bmatrix} \begin{Bmatrix} x_{c,k-2} \\ x_{s,k-2} \end{Bmatrix} \quad (53)$$

and substitute into eq. (52) to get:

$$\alpha_{k-2} = \frac{1}{2} Q_1 \frac{(x_k + \frac{1}{2} Q_1 \alpha_k)}{(x_k + \frac{1}{2} Q_1 \alpha_k)^2 + (\lambda_k + \frac{1}{2} Q_1 \beta_k)^2} \quad (54)$$

$$\beta_{k-2} = \frac{1}{2} Q_1 \frac{(\lambda_k + \frac{1}{2} Q_1 \beta_k)}{(x_k + \frac{1}{2} Q_1 \alpha_k)^2 + (\lambda_k + \frac{1}{2} Q_1 \beta_k)^2} \quad (55)$$

Noting the definition of x_k and λ_k it is seen, that as $k \rightarrow \infty$, α_k and β_k go to zero. Hence, we may choose a sufficiently high value of k that the corresponding values of α_k and β_k can be set equal to zero without notably affecting the accuracy of the calculation. Starting from this value of k , and decreasing k in steps of 2, all the α_k 's and β_k 's can be computed from the recurrence relationships above, keeping the α_k 's and β_k 's for k even separated from the α_k 's and β_k 's for k odd. Carrying the calculations out to $k=4$ and $k=3$, respectively, eqs. (50) and (51) become:

$$\begin{Bmatrix} \alpha_0 & \frac{1}{2} Q_1 & 0 \\ Q_1 & (x_2 + \frac{1}{2} Q_1 \alpha_2) & -(\lambda_2 + \frac{1}{2} Q_1 \beta_2) \\ 0 & (\lambda_2 + \frac{1}{2} Q_1 \beta_2) & (x_2 + \frac{1}{2} Q_1 \alpha_2) \end{Bmatrix} \begin{Bmatrix} x_{c0} \\ x_{c2} \\ x_{s2} \end{Bmatrix} = 0 \quad (56)$$

$$\begin{Bmatrix} (x_1 + \frac{1}{2} Q_1 (\alpha_1 + 1)) & -(\lambda_1 + \frac{1}{2} Q_1 \beta_1) \\ (\lambda_1 + \frac{1}{2} Q_1 \beta_1) & (x_1 + \frac{1}{2} Q_1 (\alpha_1 - 1)) \end{Bmatrix} \begin{Bmatrix} x_{c1} \\ x_{s1} \end{Bmatrix} = 0 \quad (57)$$

When x_{ck} and x_{sk} are different from zero, the rotor is unstable. This requires that at least one of the determinants of the two matrices vanish. Hence, the zero-point of the two determinants establish the stability boundaries. To illustrate, assume there is no damping ($B=0$, i.e. $\lambda_k=0$ and, therefore, $\beta_k=0$). Furthermore, assume that Q_1 is sufficiently small that α_1 and α_2 can be ignored (note: α is "proportional" to Q_1 , see eq. (54)). Under these assumptions the deter-

minants become:

$$x_2[x_0x_2 - \frac{1}{2}Q_1^2] = 0 \quad (58)$$

$$(x_0 + \frac{1}{2}Q_1)(x_0 - \frac{1}{2}Q_1) = 0 \quad (59)$$

Substitute for x_0 , x_1 and x_2 from eq. (48) and introduce the critical speed of the rotor:

$$\omega_c^2 = \frac{K-Q_0}{m} \quad (60)$$

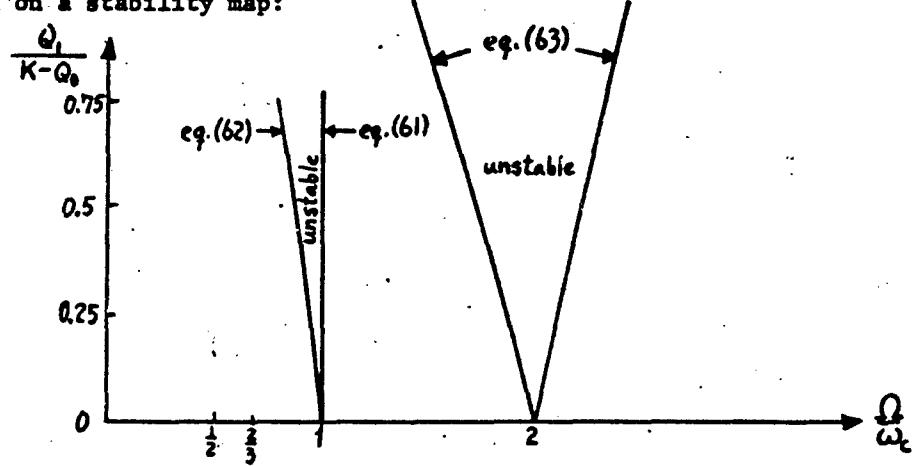
Then the determinants yield the solutions:

$$x_2 = 0 : \quad \frac{\Omega}{\omega_c} = 1 \quad (61)$$

$$x_0x_2 - \frac{1}{2}Q_1^2 = 0 : \quad \left(\frac{Q_1}{K-Q_0}\right)^2 = 2\left[1 - \left(\frac{\Omega}{\omega_c}\right)^2\right] \quad (62)$$

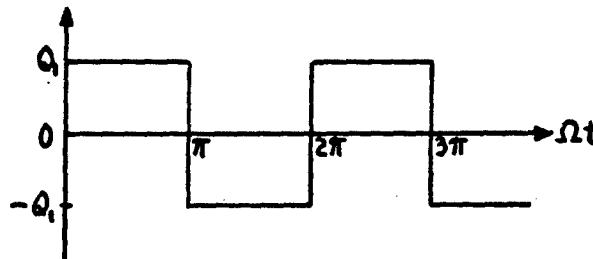
$$(x_0 + \frac{1}{2}Q_1)(x_0 - \frac{1}{2}Q_1) = 0 : \quad \frac{Q_1}{K-Q_0} = \pm 2\left[1 - \frac{1}{4}\left(\frac{\Omega}{\omega_c}\right)^2\right] \quad (63)$$

These equations define the stability boundary for small values of $Q_1/(K-Q_0)$ and for $x_3 < 0$, i.e. for $\frac{\Omega}{\omega_c} > \frac{2}{3}$. Graphically the equations can be shown on a stability map:



It is seen that the equations produce two zones of instability, one centered at $\frac{\Omega}{\omega_c} = 2$ and one at $\frac{\Omega}{\omega_c} = 1$. If more terms are carried in evaluating d_k , additional instability zones will be found, centered at $\frac{\Omega}{\omega_c} = \frac{2}{3}, \frac{3}{2}, \frac{4}{3}, \dots$, but the zones become increasingly narrower. Furthermore, when damping is added the zones no longer reach $\frac{\Omega}{(K-Q_0)} = 0$. A more detailed plot is shown in Fig. 5 where the abscissa is $\frac{\Omega}{\omega_c}$, the ordinate is $\frac{\Omega}{(K-Q_0)}$ and there are curves showing the boundaries for different values of the damping parameter $B/2\sqrt{m(K-Q_0)}$ which gives the ratio between the damping coefficient B and the critical damping. The curves are calculated on the basis of the outlined analysis with a maximum value of k of 30.

A similar stability map is shown in Fig. 6 but whereas the first map is based on a timevarying magnetic force of the form $Q_0 \cos(\Omega t)$, the second map is based on a "square wave" variation:



The difference between the two stability maps is not of any particular importance for most practical cases where $Q_0/(K-Q_0)$ seldom exceeds 1. The second map, however, illustrates that the changes in the stability zones caused by higher harmonics in the timevarying magnetic forces are small.

The analysis for calculating the stability map shown in Fig. 6 can be found in reference 1.

Turning next to the amplitude response of the rotor model considered above, assume that the rotor has a steady state eccentricity x_e' measured from the axis of the magnetic field, before the field is activated. When the magnetic field is activated, the rotor axis is pulled further out a distance x_e'' until a balance is reached between the rotor-bearing stiffness and the magnetic forces, i.e.

$$Kx''_0 = Q_0(x'_0 + x''_0)$$

i.e. $x''_0 = \frac{Q_0}{K-Q_0} x'_0$

or $x''_0 = \frac{Q_0}{K-Q_0} x'_0 + \frac{K}{K-Q_0} x''_0$ (64)

$$x''_0 = \frac{K}{K-Q_0} x''_0 \quad (65)$$

Let the rotor amplitude x be measured from this equilibrium position. Hence, the equation of motion becomes:

$$m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + K(x''_0 + x) = (Q_0 - Q_1 \cos(\Omega t))(x_0 + x)$$

or:

$$m \frac{d^2x}{dt^2} + B \frac{dx}{dt} + [K - Q_0 + Q_1 \cos(\Omega t)]x = -Q_1 x_0 \cos(\Omega t) \quad (66)$$

This equation is identical to eq. (42) except for the non-zero right hand side.

Expand x in a Fourier series:

$$x = \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] \quad (67)$$

where:

$$\psi = \Omega t \quad (68)$$

This expansion differs from the earlier one of eq. (43) by excluding all the terms with $\frac{1}{2}\Omega$ because these terms drop out in the response calculation. Hence, k is redefined and is half of its previous value. Except for this change, eq. (66) is expanded as shown earlier resulting in an infinite set of equations:

$$\begin{pmatrix} x_0 & \frac{1}{2}Q_1 & 0 & 0 & 0 & 0 & 0 & \dots \\ Q_1 & x_1 & -\lambda_1 & \frac{1}{2}Q_1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_1 & x_1 & 0 & \frac{1}{2}Q_1 & 0 & 0 & \dots \\ 0 & \frac{1}{2}Q_1 & 0 & x_2 & -\lambda_2 & \frac{1}{2}Q_1 & 0 & \dots \\ 0 & 0 & \frac{1}{2}Q_1 & \lambda_2 & x_2 & 0 & \frac{1}{2}Q_1 & \dots \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} x_{c0} \\ x_{c1} \\ x_{s1} \\ x_{c2} \\ x_{s2} \end{pmatrix} = \begin{pmatrix} 0 \\ -Q_1 x_0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \quad (69)$$

where:

$$x_k = K - Q_0 - (k\Omega)^2 m \quad (70)$$

$$\lambda_k = k\Omega B \quad (71)$$

These equations are reduced to 3 equations by the procedure employed previously to:

$$\begin{pmatrix} x_0 & \frac{1}{2}Q_1 & 0 \\ Q_1 & (x_1 + \frac{1}{2}Q_1\alpha_1) & -(\lambda_1 + \frac{1}{2}Q_1\beta_1) \\ 0 & (\lambda_1 + \frac{1}{2}Q_1\beta_1) & (x_1 + \frac{1}{2}Q_1\alpha_1) \end{pmatrix} \begin{pmatrix} x_{c0} \\ x_{c1} \\ x_{s1} \end{pmatrix} = \begin{pmatrix} 0 \\ -Q_1 x_0 \\ 0 \end{pmatrix} \quad (72)$$

The equations are readily solved to give:

$$x_{c0} = \frac{\frac{1}{2}Q_1^2(x_1 + \frac{1}{2}Q_1\alpha_1)}{x_0[(x_1 + \frac{1}{2}Q_1\alpha_1)^2 + (\lambda_1 + \frac{1}{2}Q_1\beta_1)^2] - \frac{1}{2}Q_1^2(x_1 + \frac{1}{2}Q_1\alpha_1)} \cdot x_0 \quad (73)$$

$$x_{c1} = - \frac{x_0 Q_1 (x_1 + \frac{1}{2}Q_1\alpha_1)}{x_0[(x_1 + \frac{1}{2}Q_1\alpha_1)^2 + (\lambda_1 + \frac{1}{2}Q_1\beta_1)^2] - \frac{1}{2}Q_1^2(x_1 + \frac{1}{2}Q_1\alpha_1)} \cdot x_0 \quad (74)$$

$$x_{s1} = - \frac{\lambda_1 + \frac{1}{2}Q_1\beta_1}{x_1 + \frac{1}{2}Q_1\alpha_1} \cdot x_{c1} \quad (75)$$

For simplification, assume that there is no damping ($B=0$, i.e. $\lambda_k=0$ and, therefore, $\beta_k=0$). Furthermore, assume that α_1 can be ignored. Introducing the critical speed ω_c from eq. (60) the above equations become:

$$x_{co} \approx \frac{\frac{1}{2} \left(\frac{\Omega_1}{K-Q_0} \right)^2}{1 - \left(\frac{\Omega}{\omega_c} \right)^2 - \frac{1}{2} \left(\frac{\Omega_1}{K-Q_0} \right)^2} \cdot x_0 \quad (76)$$

$$x_{c1} \approx - \frac{\frac{\Omega_1}{K-Q_0}}{1 - \left(\frac{\Omega}{\omega_c} \right)^2 - \frac{1}{2} \left(\frac{\Omega_1}{K-Q_0} \right)^2} \cdot x_0 \quad (77)$$

$$x_{s1} \approx 0 \quad (78)$$

x_{co} gives the shift in equilibrium position beyond the previously determined x_0'' , see eq. (64), such that the total static eccentricity between the magnetic axis and the rotor axis is:

$$x_0 + x_{co} \approx \frac{1 - \left(\frac{\Omega}{\omega_c} \right)^2}{1 - \left(\frac{\Omega}{\omega_c} \right)^2 - \frac{1}{2} \left(\frac{\Omega_1}{K-Q_0} \right)^2} \cdot \frac{K}{K-Q_0} \cdot x_0' \quad (79)$$

where x_0' is the mechanically built-in eccentricity.

Disregarding the frequently small term $\frac{1}{2} \left(\frac{\Omega_1}{K-Q_0} \right)^2$, the amplitude x_{c1} is the same as would be obtained if the term $Q_1 \cos(\Omega t)$ was ignored on the left hand side of eq. (66). Hence, under the stated condition the analysis can be simplified significantly.

In the preceding it has been assumed that the rotor is rigid and symmetric and, furthermore, that only one amplitude direction needs to be considered. Also, such a simple rotor model gives rise to only one resonance ("critical speed"). Although these assumptions are reasonably valid in some applications, many rotors are unsymmetric or flexible and all rotors have more than one critical speed. In addition, the timevarying magnetic forces, and frequently also the bearing stiffness and damping, cause coupling between the x and y amplitude directions. Hence, a much more extensive analysis is required to treat an arbitrary rotor. However, the

basic principle of the preceding analysis is still valid. Returning to eq. (46), which forms the basis of the previous solution, it can be written:

$$\sum_{k=0}^{\infty} [(x_k x_{ck} - \lambda_k x_{sk}) \cos(k\psi) - (\lambda_k x_{ck} + x_k x_{sk}) \sin(k\psi)] + Q_1 \cos(2\psi) \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] = 0 \quad (80)$$

where x_k and λ_k are given by eqs. (48) and (49). Introduce a complex notation:

$$x_k = x_{ck} + i x_{sk} \quad (81)$$

which is actually an abbreviated notation which in its complete form reads:

$$x_k = \operatorname{Re}\{(x_{ck} + i x_{sk}) e^{i\psi}\} = x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi) \quad (82)$$

where $\operatorname{Re}\{ \}$ means that only the real part of the bracketed expression applies. For convenience, both $\operatorname{Re}\{ \}$ and $e^{i\psi}$ are dropped during the detailed analysis and only brought back in the final answer. With this convention, eq. (80) can be written:

$$\sum_{k=0}^{\infty} (x_k + i \lambda_k) x_k + Q_1 \cos(2\psi) \sum_{k=0}^{\infty} x_k = 0 \quad (83)$$

$(x_k + i \lambda_k)$ is called the impedance of the rotor. It gives the ratio between an applied force F_{xk} with a given frequency and the resulting amplitude x_k . This is seen by simply applying a force F_{xk} with a frequency ($k \frac{\Omega}{2}$), to the rotor alone without timevarying magnetic forces. The equation of motion becomes (see eq. (42)):

$$m \frac{d^2 x_k}{dt^2} + B \frac{dx_k}{dt} + (K - Q_0) x_k = F_{xk}$$

where:

$$F_{xk} = F_{xck} + i F_{xsk} = F_{xck} \cos(k\psi) - F_{xsk} \sin(k\psi)$$

Then the solution is:

$$\frac{F_{xk}}{x_k} = [K - Q_0 - (k \frac{\Omega}{2})^2 m] + i k \frac{\Omega}{2} B = x_k + i \lambda_k \quad (84)$$

which shows the meaning of the impedance $(x_k + i \lambda_k)$.

The response at any location on an arbitrary rotor can also be represented by impedances, and these impedances are determined by applying known dynamic forces at the particular location on the rotor, computing the corresponding amplitude and taking the ratio. In an arbitrary rotor it is necessary to apply forces in both the x and y-directions so that in total:

$$F_{xk} = (x_{11} + i\lambda_{11})_k x_k + (x_{12} + i\lambda_{12})_k y_k \quad (85)$$

$$F_{yk} = (x_{21} + i\lambda_{21})_k x_k + (x_{22} + i\lambda_{22})_k y_k$$

Letting the frequency $(k \frac{\Omega}{2})$ of the applied forces take on all desired values, the impedances can be determined for $k=0, 1, 2, \dots$. These impedances can then be substituted into equations, one for the x-direction and one for the y-direction, of the same general form as eq. (46). Thereafter the corresponding infinite matrices can be formed, analogous to eqs. (50) and (51), and used in either a stability investigation or a response calculation as discussed previously.

DISCUSSION ON PERFORMING STABILITY CALCULATIONS

The general method employed in calculating the threshold of instability is discussed in the preceding chapter and the detailed analysis is given in Appendix IX. In order to make proper use of the stability computer program, described in details in Appendix XI it is necessary to be rather familiar with the analysis. Therefore, a brief discussion will be given in the following to illustrate how some knowledge of the analysis is essential to a proper interpretation of the computer output.

It should first be remarked that the computer program is written for an arbitrary rotor where, as an example, it is assumed that the bearings have different stiffnesses and damping in the vertical and horizontal directions and, furthermore, that there is coupling between the motion in the two directions. Some rotor-bearing systems are axisymmetric in which case the motions in the two directions are the same and the program does twice as many calculations as are actually necessary. Then the stability determinants tend to stay positive and the threshold of instability is not readily found because of the difficulties involved in determining those particular points where the determinants may assume a value of zero. Hence, it can be said that the program's ability to handle the completely general case sometimes makes it more difficult to examine simpler systems.

The discussion is best carried out by taking an example for illustration. The two instability determinants are given by eqs. (J.25) and (J.28), Appendix IX, as:

$$| E_{c_0} + \frac{1}{2} Q S_{c_0} + \frac{1}{2} q S_{s_0} | = 0 \quad (\text{even indices}) \quad (86)$$

$$\text{Determinant of } \left\{ (E_i + HS_i) X_i + \frac{1}{2} Q X_{c_1} + \frac{1}{2} q X_{s_1} + i(\frac{1}{2} q X_{c_1} - \frac{1}{2} Q X_{s_1}) \right\} = 0 \quad (87)$$

(odd indices)

where reference is made to Appendix IX for the meaning of the symbols. Consider a four pole homopolar generator for which the magnetic force gradients yield the two matrices (see Appendix I):

$$Q = \begin{pmatrix} 0 & 0 & 2a & 0 \\ 0 & 0 & 0 & -2a \\ 2a & 0 & 0 & 0 \\ 0 & -2a & 0 & 0 \end{pmatrix} \quad (88)$$

$$q = \begin{pmatrix} 0 & 0 & 0 & -2a \\ 0 & 0 & -2a & 0 \\ 0 & -2a & 0 & 0 \\ -2a & 0 & 0 & 0 \end{pmatrix} \quad (89)$$

where:

$$\alpha = -\frac{AB^2}{72C} L_p \quad (90)$$

Let the rotor be symmetric and such that there is no coupling between the x-amplitudes and the y-amplitudes. Hence, the rotor impedance matrices, E_k , can be written (eq. (H.72), Appendix VIII):

$$E_k = \begin{Bmatrix} (x_x + i\lambda_x)_k & 0 & 0 & 0 \\ 0 & (x_y + i\lambda_y)_k & 0 & 0 \\ 0 & 0 & (x_\phi + i\lambda_\phi)_k & 0 \\ 0 & 0 & 0 & (x_\psi + i\lambda_\psi)_k \end{Bmatrix} \quad (91)$$

If the rotor is rigid with a mass m , a transverse mass moment of inertia I and a negligible polar mass moment of inertia, the bearing stiffnesses are K_x and K_y , and the bearing dampings are B_x and B_y , the elements of the impedance matrix become:

$$\begin{aligned} x_{xk} &= 2K_x - (kv)^2 m \\ x_{yk} &= 2K_y - (kv)^2 m \\ x_{\phi k} &= \frac{1}{2} K_x l^2 - (kv)^2 I \\ x_{\psi k} &= \frac{1}{2} K_y l^2 - (kv)^2 I \\ \lambda_{xk} &= 2(kv) B_x \\ \lambda_{yk} &= 2(kv) B_y \\ \lambda_{\phi k} &= \frac{1}{2} l^2 (kv) B_x \\ \lambda_{\psi k} &= \frac{1}{2} l^2 (kv) B_y \end{aligned} \quad (92)$$

where ℓ is the rotor span between bearings and $v = \frac{1}{2} \Omega$, where Ω is the frequency of the magnetic forces.

Consider eq. (87) and restrict the analysis to a single harmonic only (i.e.

$S_1 = 0$). Substitute from eqs. (88), (89) and (91) to get:

$$\begin{Bmatrix} x_{x1} & -\lambda_{x1} & 0 & 0 & a & 0 & 0 & -a \\ \lambda_{x1} & x_{x1} & 0 & 0 & 0 & -a & -a & 0 \\ 0 & 0 & x_{y1} & -\lambda_{y1} & 0 & -a & -a & 0 \\ 0 & 0 & \lambda_{y1} & x_{y1} & -a & 0 & 0 & a \\ a & 0 & 0 & -a & x_{\theta1} & -\lambda_{\theta1} & 0 & 0 \\ 0 & -a & -a & 0 & \lambda_{\theta1} & x_{\theta1} & 0 & 0 \\ 0 & -a & -a & 0 & 0 & 0 & x_{\varphi1} & -\lambda_{\varphi1} \\ -a & 0 & 0 & a & 0 & 0 & \lambda_{\varphi1} & x_{\varphi1} \end{Bmatrix} \begin{Bmatrix} x_{c1} \\ x_{s1} \\ y_{c1} \\ y_{s1} \\ \theta_{c1} \\ \theta_{s1} \\ \varphi_{c1} \\ \varphi_{s1} \end{Bmatrix} = 0 \quad (93)$$

The determinant of this matrix is the determinant for odd indices. To evaluate the determinant it is seen that the systems of equations can be written as two sets of equations:

$$\begin{Bmatrix} (x_{x1} + i\lambda_{x1}) & 0 \\ 0 & (x_{y1} + i\lambda_{y1}) \end{Bmatrix} \begin{Bmatrix} x_{c1} + ix_{s1} \\ y_{c1} + iy_{s1} \end{Bmatrix} + \begin{Bmatrix} a & -ia \\ -ia & -a \end{Bmatrix} \begin{Bmatrix} \theta_{c1} - i\theta_{s1} \\ \varphi_{c1} - i\varphi_{s1} \end{Bmatrix} = 0 \quad (94)$$

$$\begin{Bmatrix} a & ia \\ ia & -a \end{Bmatrix} \begin{Bmatrix} x_{c1} + ix_{s1} \\ y_{c1} + iy_{s1} \end{Bmatrix} + \begin{Bmatrix} (x_{\theta1} - i\lambda_{\theta1}) & 0 \\ 0 & (x_{\varphi1} - i\lambda_{\varphi1}) \end{Bmatrix} \begin{Bmatrix} \theta_{c1} - i\theta_{s1} \\ \varphi_{c1} - i\varphi_{s1} \end{Bmatrix} = 0 \quad (95)$$

Solve eq. (95) for $\begin{Bmatrix} \theta_{c1} - i\theta_{s1} \\ \varphi_{c1} - i\varphi_{s1} \end{Bmatrix}$ and substitute into eq. (94) to get:

$$\begin{Bmatrix} (x_{x1} + i\lambda_{x1}) & 0 \\ 0 & (x_{y1} + i\lambda_{y1}) \end{Bmatrix} - \begin{Bmatrix} a & -ia \\ -ia & -a \end{Bmatrix} \begin{Bmatrix} \frac{1}{2}(x_{\theta1} - i\lambda_{\theta1}) & 0 \\ 0 & \frac{1}{2}(x_{\varphi1} - i\lambda_{\varphi1}) \end{Bmatrix} \begin{Bmatrix} a & ia \\ ia & -a \end{Bmatrix} \begin{Bmatrix} x_{c1} + ix_{s1} \\ y_{c1} + iy_{s1} \end{Bmatrix} = 0 \quad (96)$$

or:

$$\begin{Bmatrix} (x_{x1} - \xi + i\lambda_{x1}) & -i\xi \\ i\xi & (x_{y1} - \xi + i\lambda_{y1}) \end{Bmatrix} \begin{Bmatrix} x_{c1} + ix_{s1} \\ y_{c1} + iy_{s1} \end{Bmatrix} = 0 \quad (97)$$

where:

$$\xi = \alpha^2 \left[\frac{1}{x_{x_1} - i\lambda_{x_1}} + \frac{1}{x_{y_1} + i\lambda_{y_1}} \right] = \alpha^2 \left[\frac{x_{y_1} + i\lambda_{y_1}}{x_{x_1}^2 + \lambda_{x_1}^2} + \frac{x_{x_1} + i\lambda_{x_1}}{x_{y_1}^2 + \lambda_{y_1}^2} \right] \quad (98)$$

The determinant becomes:

$$\Delta_{\text{det}} = (x_{x_1} - \xi + i\lambda_{x_1})(x_{y_1} - \xi + i\lambda_{y_1}) - \xi^2 = (x_{x_1} + i\lambda_{x_1})(x_{y_1} + i\lambda_{y_1}) - \xi(x_{x_1} + i\lambda_{x_1} + x_{y_1} + i\lambda_{y_1}) \quad (99)$$

which is zero for:

$$\xi = \left[\frac{1}{x_{x_1} + i\lambda_{x_1}} + \frac{1}{x_{y_1} + i\lambda_{y_1}} \right]^{-1} \quad (100)$$

or for:

$$\alpha^2 = \left[\frac{1}{x_{x_1} + i\lambda_{x_1}} + \frac{1}{x_{y_1} + i\lambda_{y_1}} \right]^{-1} \left[\frac{1}{x_{x_1} - i\lambda_{x_1}} + \frac{1}{x_{y_1} - i\lambda_{y_1}} \right]^{-1} \quad (101)$$

Assume that the bearings have no damping, i.e. $\lambda_{x_1} = \lambda_{y_1} = \lambda_{x_1} = \lambda_{y_1} = 0$, whereby eq. (101) becomes:

$$\alpha^2 = \frac{x_{x_1} x_{y_1} x_{x_1} x_{y_1}}{(x_{x_1} + x_{y_1})(x_{x_1} + x_{y_1})} \quad (102)$$

The rotor has four critical speeds:

$$\begin{aligned} \omega_x &= \sqrt{\frac{2K_x}{m}} \\ \omega_y &= \sqrt{\frac{2K_y}{m}} \\ \omega_x &= \sqrt{\frac{\frac{1}{4}l^2 K_x}{m}} \\ \omega_y &= \sqrt{\frac{\frac{1}{4}l^2 K_y}{m}} \end{aligned} \quad (103)$$

Substitute from eqs. (92) into eq. (102), making use of eqs. (103):

$$\frac{a^2}{L^2 K_x K_y} = \frac{[1 - (\frac{\omega}{\omega_x})^2][1 - (\frac{\omega}{\omega_y})^2][1 - (\frac{\omega}{\omega_p})^2][1 - (\frac{\omega}{\omega_q})^2]}{[1 + (\frac{\omega_x}{\omega_y})^2 - 2(\frac{\omega}{\omega_y})^2][1 + (\frac{\omega_p}{\omega_q})^2 - 2(\frac{\omega}{\omega_q})^2]} \quad (104)$$

Since it is a four pole homopolar generator, $\nu = \omega$ where ω is the angular speed of the rotor.

The stability map defined by eq. (104) is best illustrated by assuming certain numerical values. Let $K_y = \frac{1}{2} K_x$ (i.e. the bearings are twice as stiff in the x-direction as in the y-direction). Then:

$$\left(\frac{\omega_x}{\omega_y}\right)^2 = \left(\frac{\omega_p}{\omega_q}\right)^2 = \frac{K_x}{K_y} = 2$$

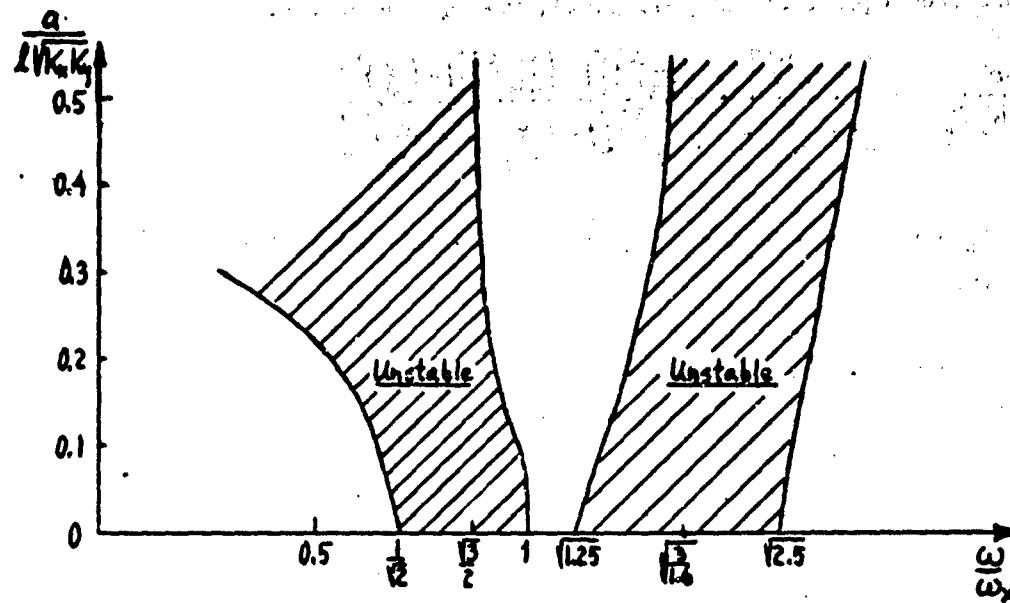
Assume also, that the conical critical speed, ω_0 , is 1.58 times the translatory critical speed ω_x such that:

$$\left(\frac{\omega_0}{\omega_x}\right)^2 = 2.5$$

Then:

$$\frac{a^2}{L^2 K_x K_y} = \frac{[1 - (\frac{\omega}{\omega_x})^2][1 - 2(\frac{\omega}{\omega_x})^2][1 - 0.4(\frac{\omega}{\omega_x})^2][1 - 0.8(\frac{\omega}{\omega_x})^2]}{9[1 - \frac{1}{3}(\frac{\omega}{\omega_x})^2][1 - \frac{1.6}{3}(\frac{\omega}{\omega_x})^2]}$$

The corresponding stability plot becomes:



When it is now considered that there will be analogous instability zones located at $\frac{\omega}{\omega_x} = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, etc. and, furthermore, for $\frac{\omega}{\omega_x} > 1$ there may be instability zones caused by the higher critical speeds of the rotor, it is readily seen that the complete stability map can easily become very complicated. However, it is necessary to have some preconcept of where the instability zones are, otherwise it is easy to misinterpret the results from the computer program. Thus, if in the above example the rotor is operating at $\frac{\omega}{\omega_x} = 1.5$, the stability determinant would never be equal to zero because the rotor is inherently unstable and the determinant only indicates the threshold of instability, i.e. the boundaries of the instability zones. It cannot tell if the rotor is stable or unstable.

When bearing damping is present, the instability zones move up in the above map and some of the zones may actually disappear altogether. It is still recommended to perform a calculation without damping first in order to locate the potential instability zones. It is then easier to decide where to search for the instability threshold when damping is included.

The computer program searches for the threshold of instability by varying the value of the magnetic force gradient. In other words, in terms of the above example, α is varied over a specified range at a fixed value of ω/ω_x and for each value of α the two determinants are computed. The program detects if any of the determinants changes sign but is, of course, otherwise unable to decide if a determinant has a zero-point. The numerical round-off errors and also the fact that the determinants are only evaluated at discrete values, usually prevent the detection of a zero-point where the slope of the determinant is zero. For this reason it is frequently necessary to let the magnetic force gradient vary in fine increments.

In summary, the recommended procedure for performing a stability calculation is:

a. Determine all the critical speeds of the rotor that may possibly influence the stability of the rotor. These are the critical speeds which are close to $1/2$, 1 and maybe even $3/2$ times the frequency of the magnetic forces (it depends on how well they are damped). The two lowest critical speeds should always be included and frequently also the third critical speed.

b. The magnetic force frequency, Ω , is a fixed ratio of the rotor speed ω . For the four pole homopolar generator, $\frac{\Omega}{\omega} = 2$ and for the heteropolar inductor generator, $\frac{\Omega}{\omega}$ is equal to the number of rotor teeth. Then, on the basis of the known critical speeds select two rotor speeds, one on each side of the operating speed. These two speeds are determined as the speeds closest to the operating speed from the following relationships:

$$\omega = \begin{cases} 2 \\ 1 \\ \frac{2}{3} \end{cases} \frac{\omega_{\text{critical}}}{(\frac{\Omega}{\omega})}$$

To illustrate, assume that a four pole homopolar generator operates at 12,000 rpm ($\frac{\Omega}{\omega} = 2$) and that its first three critical speeds are at 9,000, 11,000 and 32,000 rpm. Then the rotor speeds at which the rotor is susceptible to instability are:

9,000 rpm	11,000 rpm	32,000 rpm
4,500 rpm	5,500 rpm	16,000 rpm
3,000 rpm	3,700 rpm	10,700 rpm

Hence, the minimum speed range for the calculations are from 11,000 to 16,000 rpm.

c. Perform stability calculations covering the determined speed range and leaving out any bearing damping. In this way, a stability map for the undamped system is obtained. If the actual magnetic force gradients are such that the rotor operates in a stable zone, the rotor is stable and no further calculations are required. Otherwise, perform additional calculations in which the bearing damping is included. In these calculations the magnetic force gradients should be varied in very small steps in the neighborhood of the threshold in order to determine the exact zero-point (or minimum point) of the instability determinant. If the rotor operates below the zero-point it is stable, otherwise unstable (in theory there are exceptions to this rule but in practice the rule should be valid).

DISCUSSION ON PERFORMING RESPONSE CALCULATIONS

A rotor response calculation is considerably simpler to perform than a stability calculation and does not require the same understanding of the detailed analysis. However, some knowledge of the analysis may prove helpful in certain cases.

Let the frequency of the magnetic forces be Ω and the angular speed of the rotor is ω . The ratio: $\frac{\Omega}{\omega}$ is fixed for a given generator ($\frac{\Omega}{\omega} = 2$ for the 4 pole homopolar generator, and $\frac{\Omega}{\omega}$ is equal to the number of teeth for the heteropolar inductor generator). The rotor is forced to whirl by the magnetic forces produced when the rotor axis does not coincide with the magnetic axis of the alternator stator. The position of the rotor axis is defined by four coordinates: the eccentricity components x_0 and y_0 measured in the center-plane of the alternator, and the misalignment angles θ_0 and ϕ_0 . In the homopolar generator, both forces and moments will be set up such that the forces are proportional to θ_0 and ϕ_0 , and the moments are proportional to x_0 and y_0 . In the heteropolar inductor generator, only forces are produced. They are proportional to x_0 and y_0 .

The fundamental response of the rotor has the same frequency as the magnetic forces (i.e. the amplitudes vary harmonically with the frequency Ω). In addition, higher harmonics of the fundamental frequency will also be excited which means that the vibratory response will contain components not only with the fundamental frequency Ω but also with frequencies 2Ω , 3Ω , and so on. However, the excitation force available for the higher harmonics normally decrease rapidly with the number of the harmonics. Let the gradients of the magnetic forces be represented by the symbol a and let the combined rotor-bearing stiffness be represented by K . If the number of the harmonic is n , the available excitation force for that harmonic is very roughly proportional to $(\frac{a}{K})^{(2n-1)}$. Thus, if the magnetic force gradient is, say 30 percent of the rotor-bearing stiffness (i.e. $\frac{a}{K} = 0.3$) and resonance effects are ignored, the amplitudes of the second harmonic are of the order of 10 percent of the amplitudes of the fundamental harmonic, and the amplitudes of the third harmonic are only of the order of 1 percent of the fundamental harmonic. Hence, it is readily seen that

unless $\frac{g}{K}$ is reasonably large, only the fundamental harmonic, or possibly the two first harmonics, are of any practical significance. The only possible exception is when one of the harmonic frequencies is close to a resonance of the rotor-bearing system for which little damping is provided. In that case, even if the excitation force may be small, the corresponding amplitudes could become appreciable. The resonant peak, on the other hand, will be very narrow. It is, therefore, recommended that when a rotor response calculation is performed, the calculation is not limited just to the operating speed but covers a reasonable speed range around the operating speed. In this way it will be possible to detect if there are any high amplitude response too close to the operating speed.

SUMMARY

The principal objectives of this volume are: a) to give formulas from which the magnetic forces in three representative generator types can be calculated, b) to provide a computer program to calculate the stability of the alternator rotor, and c) to provide a computer program to calculate the amplitude response of the alternator rotor. It is the intention that these engineering tools can be used in future design and development work in the application of alternators to space power plants and similar machinery, and both the formulas and the two computer programs are presented in a form where they can be readily applied to an actual application.

To establish the formulas and the computational methods, a rather complex analysis has been performed. There is no previous work in this field on which the analysis can be based and it is believed that several of the developed methods may be of value in future work on electromagnetic force interaction and rotor dynamics.

At present there is little test experience or experimental data against which the results of this investigation can be compared. For this reason and, more significantly, also because it is a problem of serious practical concern, it would be desirable to perform a similar investigation of the effect of the magnetic forces on the rotor of an electrical motor. As mentioned previously, severe vibration problems have been encountered in at least three electrical motor applications and it would be of importance to determine the exact causes of the vibrations so that the problem may be avoided in future motor designs. The methods presented in this volume could serve as a basis for such an investigation.

ACKNOWLEDGMENT

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FIGURES

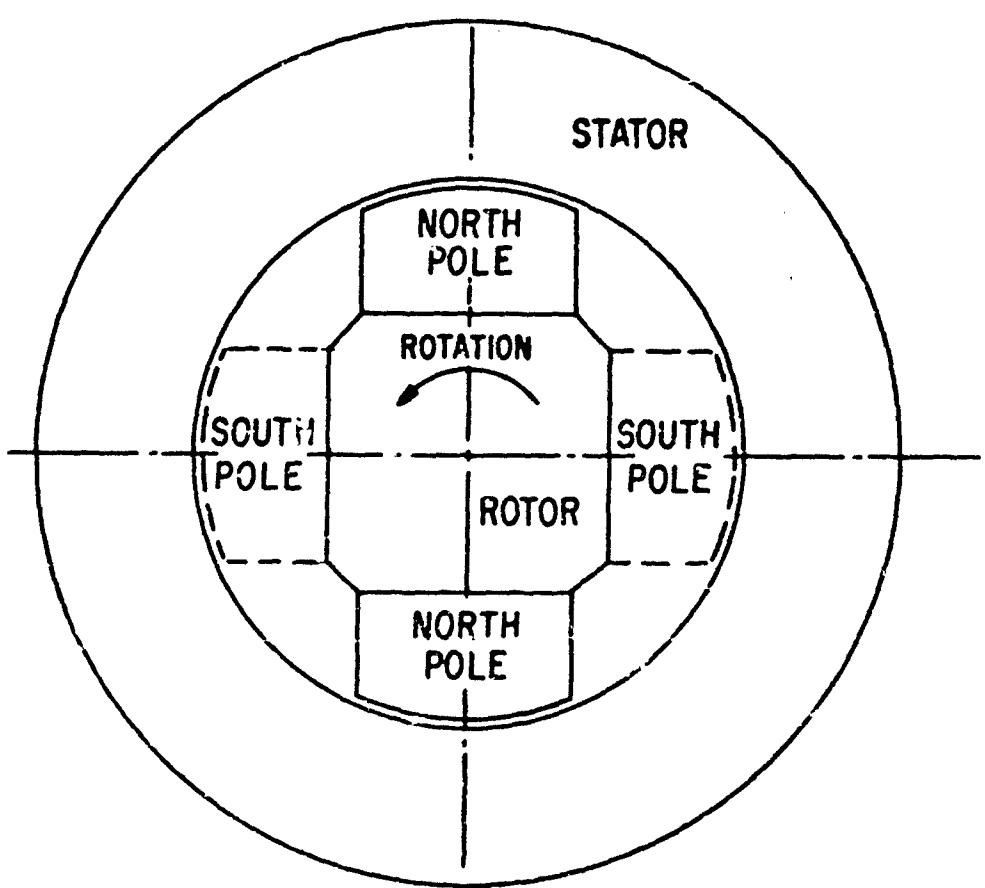
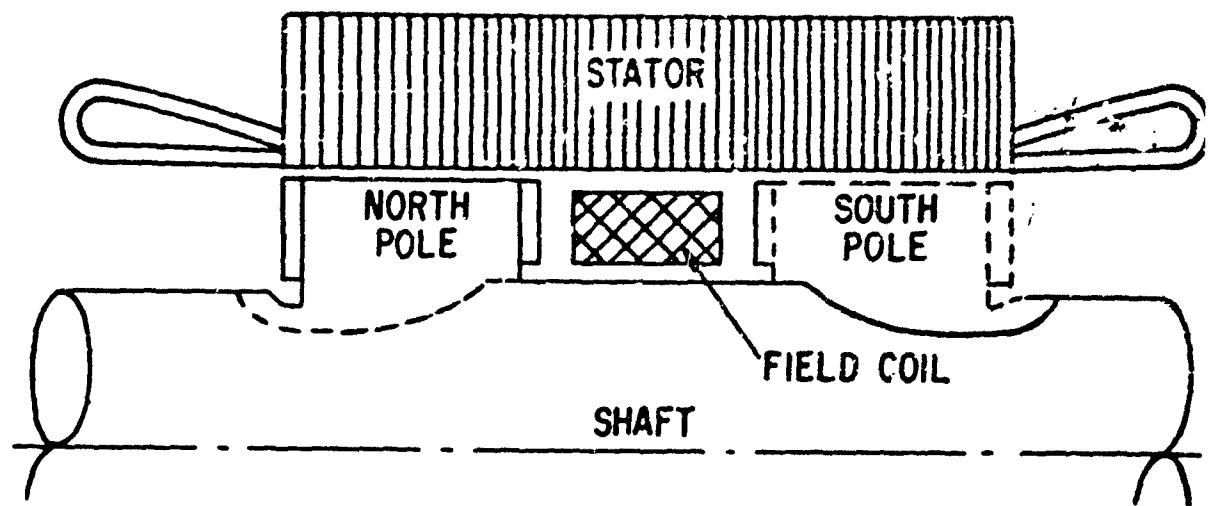


Figure 1 Homopolar Generator

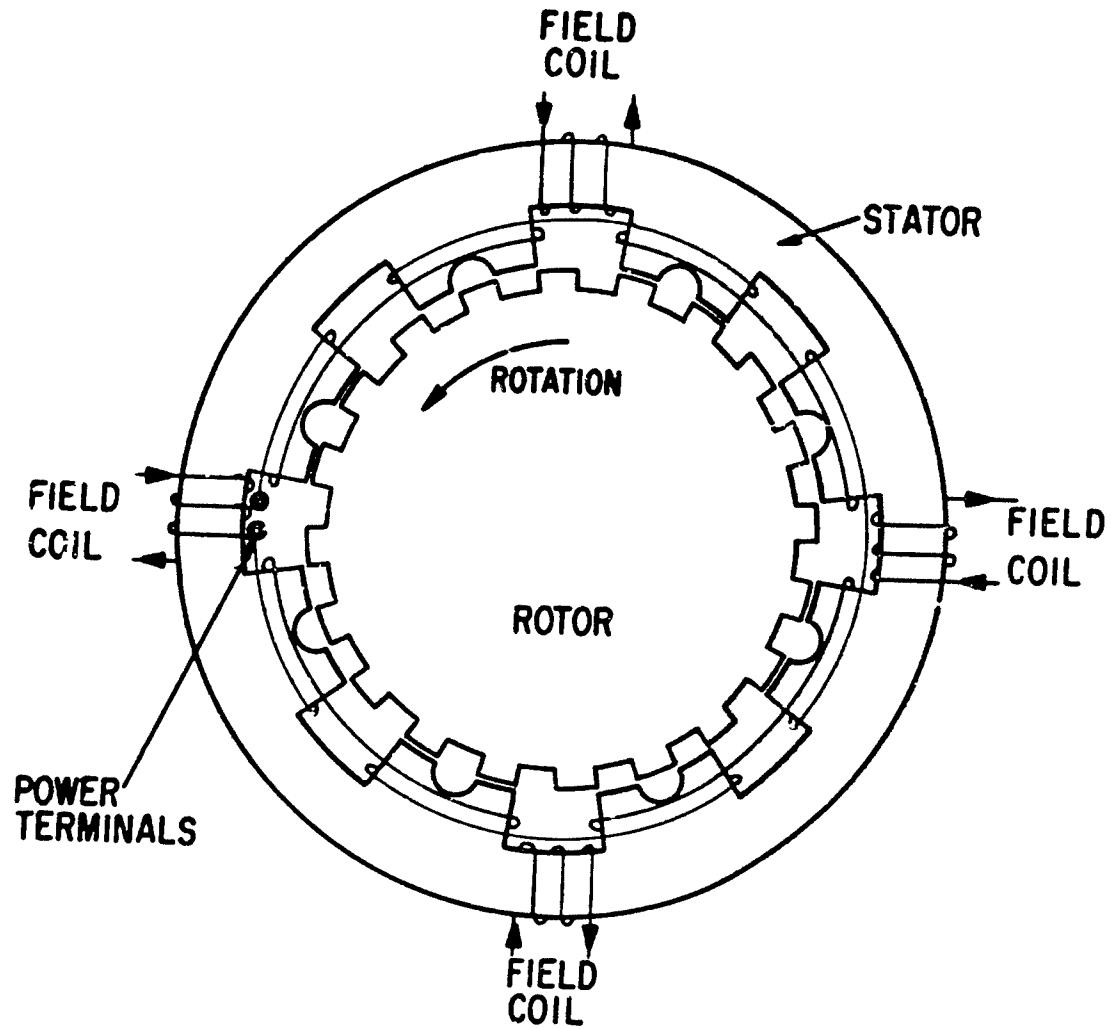
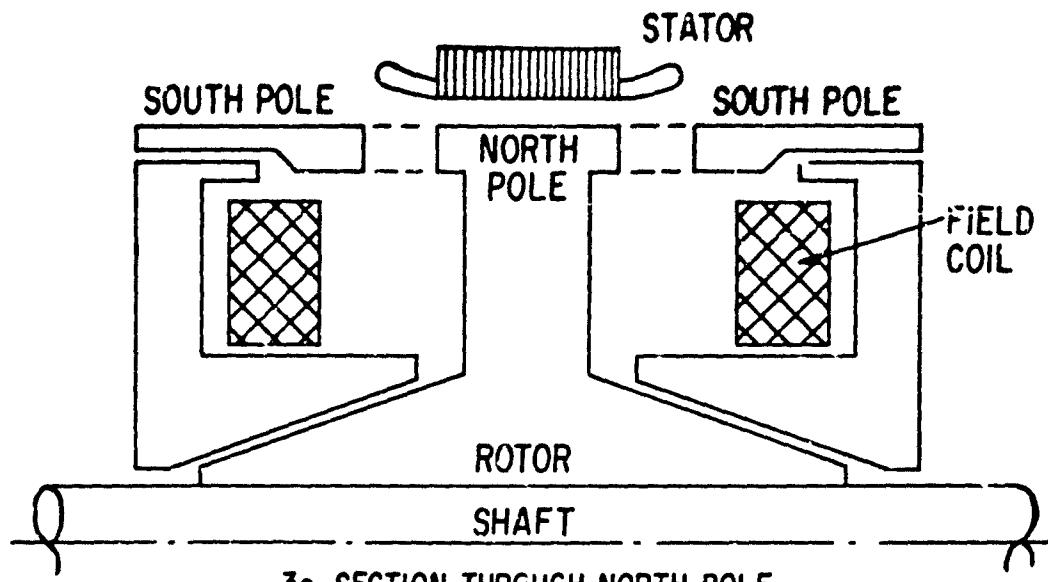
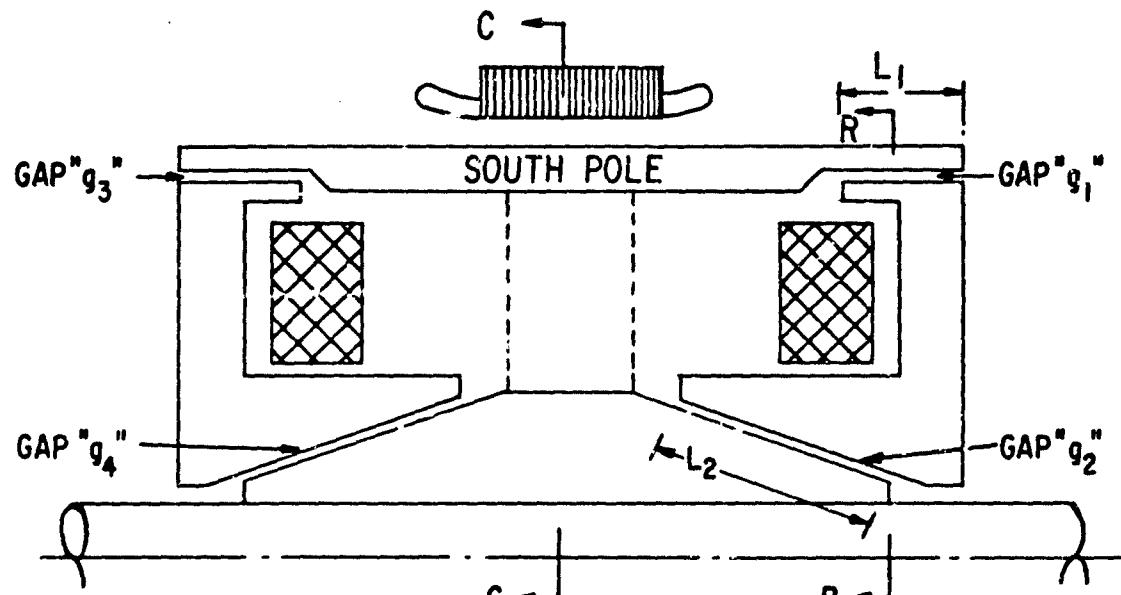


Figure 2 Heteropolar Generator

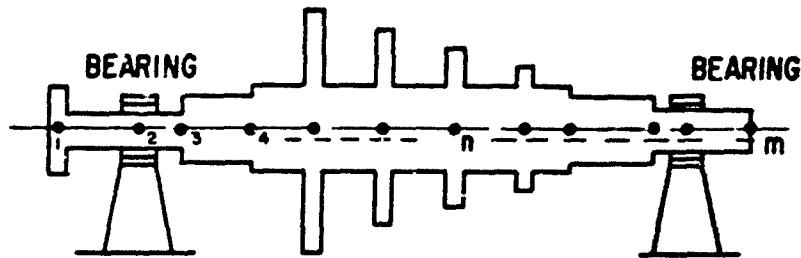


3a SECTION THROUGH NORTH POLE

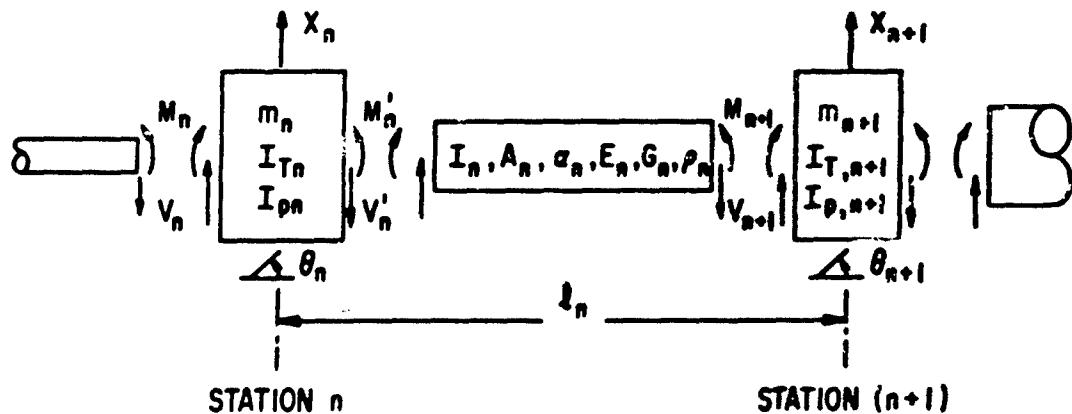


3b SECTION THROUGH SOUTH POLE

Figure 3 Two-Coil Lundell Generator



Outline of Rotor with Location of Rotor Stations



Sign Convention for Amplitude, Slope, Bending Moment and Shear Force

Figure 4 Rotor Model and Sign Convention for Analysis of Rotor Impedance

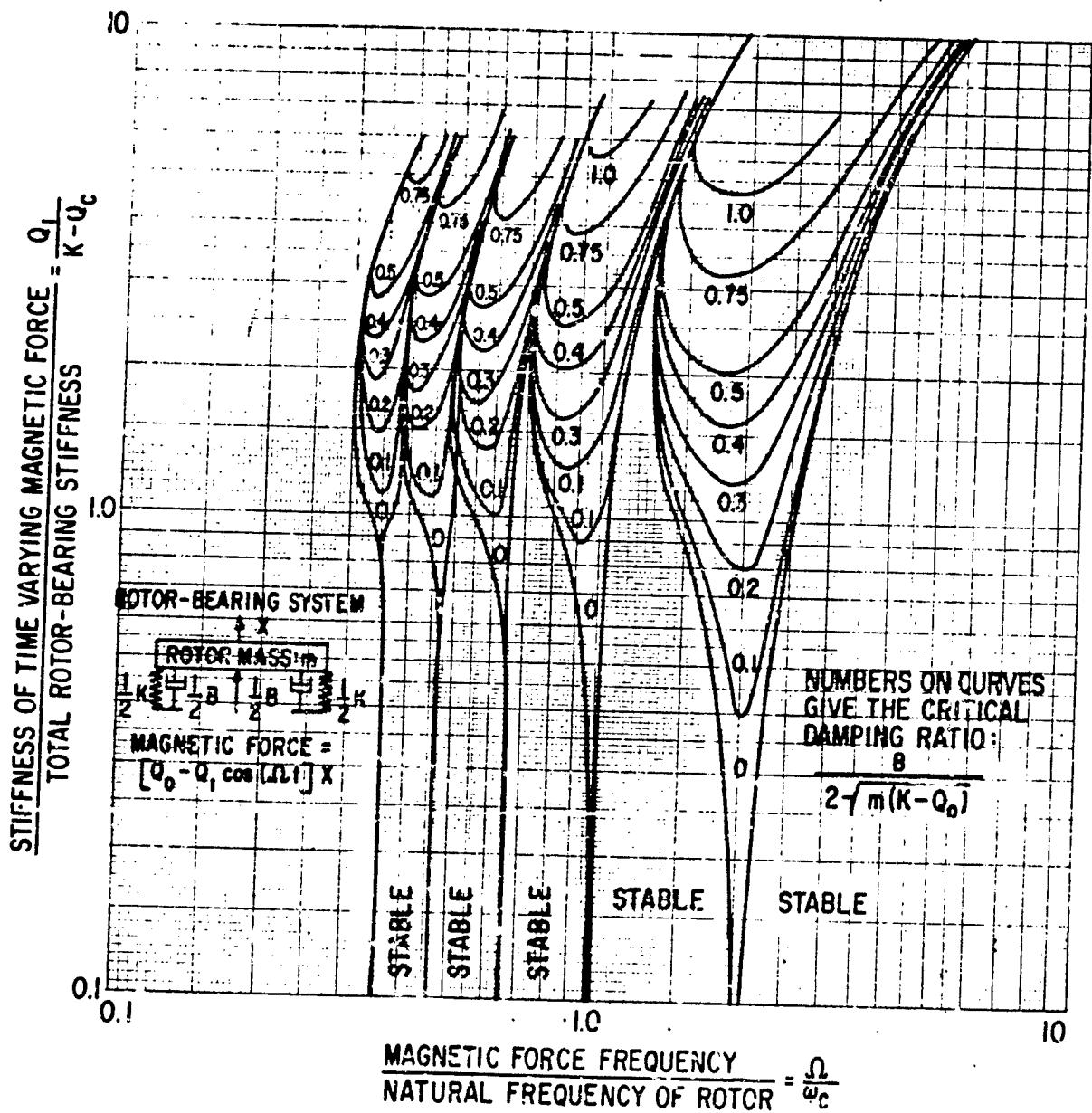


Figure 5 Stability Map For A Rigid Rotor With A Harmonically Varying Magnetic Force

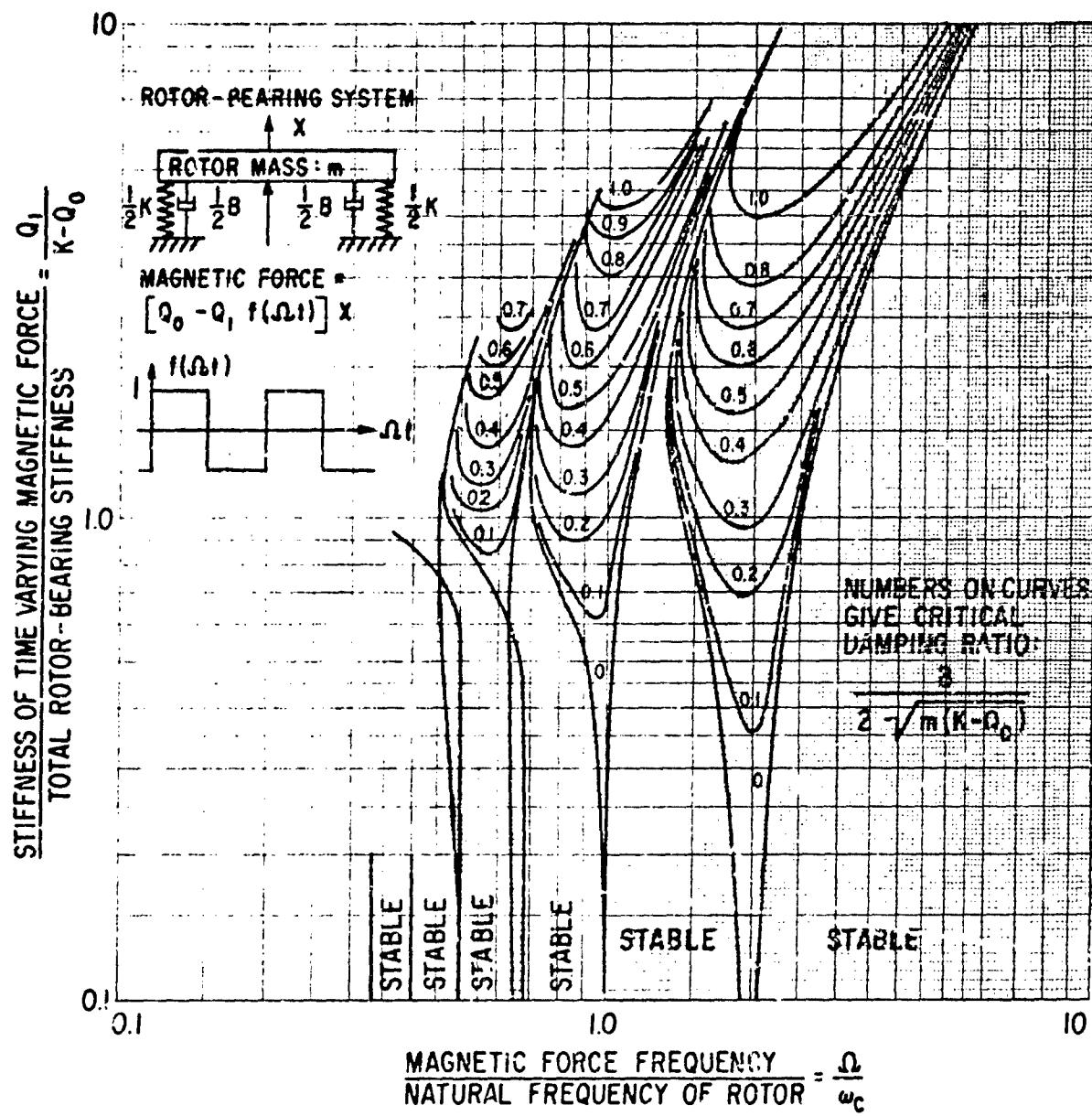


Figure 6 Stability Map For A Rigid Rotor With A "Square Wave" Varying Magnetic Force

APPENDIX I: Magnetic Forces of a Homopolar Generator Operating With No Load

A homopolar generator is shown schematically in Fig. 1. It is a brushless generator whose field coil is located between the plane of the north poles and the plane of the south poles. Let there be n north poles and n southpoles. Furthermore, the field coil has N_f windings such that it's mmf, \mathcal{J}_f is:

$$\mathcal{J}_f = N_f i_f \quad (A.1)$$

where i_f is the current in the coil.

The magnetic reluctance of the airgap at the k' th north pole is R_{Nk} and at the k' th south pole R_{Sk} . Set:

$$\frac{1}{R_N} = \sum_{k=1}^n \frac{1}{R_{Nk}} \quad (A.2)$$

$$\frac{1}{R_S} = \sum_{k=1}^n \frac{1}{R_{Sk}} \quad (A.3)$$

The flux, Φ leaving the rotor through the north poles is the same flux that enters the rotor through the south poles. When the mmf's across the airgaps of the north poles and the south poles are $\mathcal{J}_N = \mathcal{J}_S$, respectively, the equations relating flux and mmf becomes:

$$\Phi = \frac{\mathcal{J}_N}{R_N} = \frac{\mathcal{J}_S}{R_S} \quad (A.4)$$

Since R_N and R_S are in series, the total reluctance of the flux path is $(R_N + R_S)$, ignoring the reluctance of the iron (i.e. saturation effects are ignored). Hence:

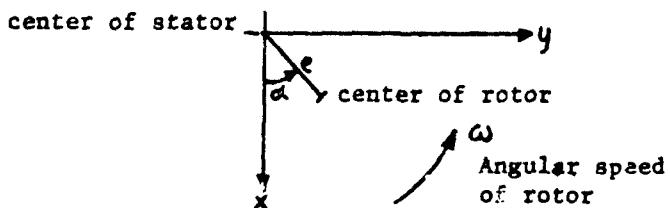
$$\Phi = \frac{\mathcal{J}_f}{R_N + R_S} \quad (A.5)$$

Combining eqs. (A.4) and (A.5):

$$\mathcal{J}_N = \frac{R_N}{R_N + R_S} \mathcal{J}_f \quad (A.6)$$

$$\mathcal{J}_S = \frac{R_S}{R_N + R_S} \mathcal{J}_f \quad (A.7)$$

To determine the reluctances, assume the rotor to be eccentric by the distance e from the center of the stator and let the angle between the direction of displacement and the vertical axis (the x -axis) be α :



Define the eccentricity ratio ϵ by:

$$\epsilon = \frac{e}{C} \quad (\text{A.8})$$

where C is the mean radial gap at the poles. Hence:

$$x = e \cos \alpha = C \epsilon \cos \alpha \quad (\text{A.9})$$

$$y = e \sin \alpha = C \epsilon \sin \alpha \quad (\text{A.10})$$

At time $t=0$, the first northpole is on the x -axis. Hence the center of the k' th pole is at an angle $(k-1) \frac{2\pi}{n}$ from the x -axis at $t=0$. When the angular speed of the rotor is ω , the airgap at the center of the k' th northpole can be expressed as:

$$h_{Nk} = C \left[1 - \epsilon \cos \left(\omega t - \alpha + \frac{2\pi}{n} (k-1) \right) \right] \quad (\text{A.11})$$

The southpoles are displaced $\frac{\pi}{n}$ from the northpoles, and the airgap at the center of the k' th southpole becomes:

$$h_{Sk} = C \left[1 - \epsilon \cos \left(\omega t - \alpha + \frac{\pi}{n} + \frac{2\pi}{n} (k-1) \right) \right] \quad (\text{A.12})$$

Thus, the reluctance of the airgap of the k' th northpole becomes:

$$R_{Nk} = \frac{h_{Nk}}{A\mu} = \frac{C}{A\mu} \left[1 - \epsilon \cos \left(\omega t - \alpha + \frac{2\pi}{n} (k-1) \right) \right] \quad (\text{A.13})$$

where A is the area of a pole and μ is the permeability. If ϵ is assumed small ($\epsilon \ll 1$), eq. (A.13) yields:

$$\frac{1}{R_{Nk}} = \frac{A\mu}{C} \left[1 + \epsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right] \quad (\text{A.14})$$

Hence, from eq. (A.2):

$$\begin{aligned} \frac{1}{R_N} &= \frac{A\mu}{C} \left[n + \epsilon \sum_{k=1}^n \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right] \\ &= \frac{A\mu}{C} \left[n + \epsilon \cos(\omega t - \alpha) \sum_{k=1}^n \cos\left(\frac{2\pi}{n}(k-1)\right) - \epsilon \sin(\omega t - \alpha) \sum_{k=1}^n \sin\left(\frac{2\pi}{n}(k-1)\right) \right] \end{aligned}$$

Now:

$$\sum_{k=1}^n \cos\left(k \frac{2\pi}{n}\right) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n \geq 2 \end{cases} \quad (\text{A.16})$$

$$\sum_{k=1}^n \sin\left(k \frac{2\pi}{n}\right) = 0 \quad (\text{A.17})$$

Ignoring the case of $n=1$, where there are no magnetic forces anyway, eq. (A.15) becomes:

$$R_N = \frac{C}{nA\mu} \quad (\text{A.18})$$

Similarly, it is found that:

$$R_S = \frac{C}{nA\mu} \quad (\text{A.19})$$

whereby eqs. (A.6) and (A.7) yield:

$$\mathcal{I}_N = \mathcal{I}_S = \frac{1}{2} \mathcal{I}_f \quad (\text{A.20})$$

Since both \mathcal{I}_N and \mathcal{I}_S , and also R_N and R_S are independent of time, the total flux Φ will also be independent of time (see eq. (A.4)) which means that there is no self-induced current in the field coil.

On this basis the flux for the k'th northpole becomes:

$$\Phi_{Nk} = \frac{3_s}{R_{Nk}} = \frac{3_s A \mu}{2C} \left[1 + \epsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right] \quad (\text{A.21})$$

and for the k'th southpole:

$$\Phi_{Sk} = \frac{3_s}{R_{Sk}} = \frac{3_s A \mu}{2C} \left[1 + \epsilon \cos(\omega t - \alpha + \frac{\pi}{n} + \frac{2\pi}{n}(k-1)) \right] \quad (\text{A.22})$$

At each pole there is a radial force pulling on the rotor. For the k'th northpole this force becomes:

$$\bar{Q} \left(\frac{\Phi_{Nk}}{A} \right)^2$$

where \bar{Q} is a constant which depends on the units employed for the quantities. If Φ is in lines, A is in inch² and the force is measured in lbs., then

$$\bar{Q} = \frac{1}{72,130,000}$$

This force has x and y components which for the k'th northpole become:

$$(F_{Nx})_k = \bar{Q} \frac{\Phi_{Nk}^2}{A} \cos(\omega t + \frac{2\pi}{n}(k-1)) \quad (\text{A.23})$$

$$(F_{Ny})_k = \bar{Q} \frac{\Phi_{Nk}^2}{A} \sin(\omega t + \frac{2\pi}{n}(k-1)) \quad (\text{A.24})$$

When Φ_{Nk} is substituted from eq. (A.21) and the forces are summed over all n northpoles, the total forces acting on the rotor in the plane of the northpoles become:

$$F_{Nx} = \sum_{k=1}^n (F_{Nx})_k = \bar{Q} \frac{3_s^2 A \mu^2}{4C} \sum_{k=1}^n \left[1 + 2\epsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right] \cos(\omega t + \frac{2\pi}{n}(k-1)) \quad (\text{A.25})$$

$$F_{Ny} = \sum_{k=1}^n (F_{Ny})_k = \bar{Q} \frac{3_s^2 A \mu^2}{4C} \sum_{k=1}^n \left[1 + 2\epsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1)) \right] \sin(\omega t + \frac{2\pi}{n}(k-1)) \quad (\text{A.26})$$

By expanding the trigonometric functions and making use of eqs. (A.16) and (A.17), these equations can be written:

$$F_{px} = \bar{Q} \frac{3^2 A \mu}{4 C^2} \epsilon \sum_{k=1}^n \left\{ \cos \alpha [1 + \cos 2(\omega t + \frac{2\pi}{n}(k-1))] + \sin \alpha \sin 2(\omega t + \frac{2\pi}{n}(k-1)) \right\} \quad (\text{A.27})$$

$$F_{Ny} = \bar{Q} \frac{3^2 A \mu}{4 C^2} \epsilon \sum_{k=1}^n \left\{ \cos \alpha \cdot \sin 2(\omega t + \frac{2\pi}{n}(k-1)) + \sin \alpha [1 - \cos 2(\omega t + \frac{2\pi}{n}(k-1))] \right\} \quad (\text{A.28})$$

The following identities hold true:

$$\sum_{k=1}^n \cos(k \frac{4\pi}{n}) = \begin{cases} 1 & \text{for } n=1 \\ 2 & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases} \quad (\text{A.29})$$

$$\sum_{k=1}^n \sin(k \frac{4\pi}{n}) = 0 \quad \text{for all } n \quad (\text{A.30})$$

Furthermore, since the total flux is Φ the average flux density B_0 is:

$$B_0 = \frac{\Phi}{nA} = \frac{\bar{J}_f}{nA(R_N + R_S)} = \frac{\mu \bar{J}_f}{2C} \quad (\text{A.31})$$

Introducing these equations into eqs. (A.27) and (A.28) and making use of eqs. (A.9) and (A.10), the result becomes:

For $n=2$

$$F_{Nx} = 2\bar{Q} \frac{AB_0^2}{C} [x_N(1 + \cos(2\omega t)) + y_N \sin(2\omega t)] \quad (\text{A.32})$$

$$F_{Ny} = 2\bar{Q} \frac{AB_0^2}{C} [x_N \sin(2\omega t) + y_N(1 - \cos(2\omega t))] \quad (\text{A.33})$$

For $n \geq 3$

$$F_{Nx} = n\bar{Q} \frac{AB_0^2}{C} x_N \quad (\text{A.34})$$

$$F_{Ny} = n\bar{Q} \frac{AB_0^2}{C} y_N \quad (\text{A.35})$$

Similarly, the forces in the plane of the southpoles become:

For $n=2$

$$F_{sx} = 2\bar{Q} \frac{AB_0^2}{C} [x_s(1-\cos(2\omega t)) - y_s \sin(2\omega t)] \quad (\text{A.36})$$

$$F_{sy} = 2\bar{Q} \frac{AB_0^2}{C} [-x_s \sin(2\omega t) + y_s(1+\cos(2\omega t))] \quad (\text{A.37})$$

For $n \geq 3$

$$F_{sx} = n\bar{Q} \frac{AB_0^2}{C} x_s \quad (\text{A.38})$$

$$F_{sy} = n\bar{Q} \frac{AB_0^2}{C} y_s \quad (\text{A.39})$$

For use in the stability and response calculation, these forces should be written in a different form. Let the distance between the pole planes be L_p and let the rotor displacement in the center between the two planes be x and y . Furthermore, let the rotor have the slopes $\theta = \frac{dx}{dz}$ and $\phi = \frac{dy}{dz}$ where z is the axial coordinate. Then the displacements in the pole planes become:

$$x_N = x + \frac{1}{2}L_p\theta \quad y_N = y + \frac{1}{2}L_p\phi \quad (\text{A.40})$$

$$x_s = x - \frac{1}{2}L_p\theta \quad y_s = y - \frac{1}{2}L_p\phi$$

The forces and moments acting on the rotor become:

$$F_x = F_{Nx} + F_{sx} \quad F_y = F_{Ny} + F_{sy} \quad (\text{A.41})$$

$$T_x = \frac{1}{2}L_p(F_{Nx} - F_{sx}) \quad T_y = \frac{1}{2}L_p(F_{Ny} - F_{sy})$$

Substitute eqs. (A.40) into eqs. (A.32) to (A.39) and combine them according to eq. (A.41) to get:

For $n=2$

$$F_x = 2\bar{Q} \frac{AB_0^2}{C} [2x + \theta L_p \cos(2\omega t) + \phi L_p \sin(2\omega t)] \quad (\text{A.42})$$

$$F_y = 2\bar{Q} \frac{AB_0^2}{C} [2y + \theta L_p \sin(2\omega t) - \phi L_p \cos(2\omega t)] \quad (\text{A.43})$$

$$T_x = 2\bar{Q} \frac{AB_0^2}{C} [x L_p \cos(2\omega t) + y L_p \sin(2\omega t) + \frac{1}{2}L_p^2 \theta] \quad (\text{A.44})$$

$$T_y = 2\bar{Q} \frac{AB_0^2}{C} [x L_p \sin(2\omega t) - y L_p \cos(2\omega t) + \frac{1}{2}L_p^2 \phi] \quad (\text{A.45})$$

which can be written in matrix form:

$$\text{For } n=2 \quad \begin{cases} F_x \\ F_y \\ T_x \\ T_y \end{cases} = \begin{cases} Q_0 x \\ Q_0 y \\ Q'_0 \theta \\ Q'_0 \phi \end{cases} - [Q \cos(2\omega t) - q \sin(2\omega t)] \begin{cases} x \\ y \\ \theta \\ \phi \end{cases} \quad (\text{A.46})$$

Here:

For $n=2$

$$Q_0 = 4 \bar{Q} \frac{AB_o^2}{C} \quad \frac{\text{lbs}}{\text{inch}} \quad (\text{A.47})$$

$$Q'_0 = \bar{Q} \frac{AB_o^2}{C} L_p^2 \quad \frac{\text{lbs}\cdot\text{inch}}{\text{radian}} \quad (\text{A.48})$$

and Q and q are 4 by 4 matrices:

$$\text{For } n=2 \quad Q = 2 \bar{Q} \frac{AB_o^2}{C} L_p \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{lbs} \quad (\text{A.49})$$

$$q = 2 \bar{Q} \frac{AB_o^2}{C} L_p \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{lbs} \quad (\text{A.50})$$

In this form the results can be used directly in the stability and response calculations. When there are more than four poles (i.e. $n \geq 3$) the forces are not time dependent and, hence, a stability or response calculation of the type under investigation does not apply. However, there will still be negative lateral and moment stiffnesses which must be taken into account when performing the more conventional rotor unbalance response calculation.

These spring coefficients are:

For $n \geq 3$

$$Q_0 = 2n \bar{Q} \frac{AB_o^2}{C} \quad \frac{\text{lbs}}{\text{inch}}$$

$$Q'_0 = \frac{1}{2} n \bar{Q} \frac{AB_o^2}{C} L_p^2 \quad \frac{\text{lbs}\cdot\text{inch}}{\text{radian}}$$

APPENDIX II: Magnetic Forces of a Heteropolar Inductor Generator Operating with No Load

A cross-section of a heteropolar generator is shown schematically in Fig. 2. It is a brushless generator with a field coil for each pole such that a pole receives its flux from two field coils. Each pole has two faces which are provided with teeth. The rotor likewise has teeth. Schematically, with 2 north poles and 2 south poles the magnetic circuit can be shown as:

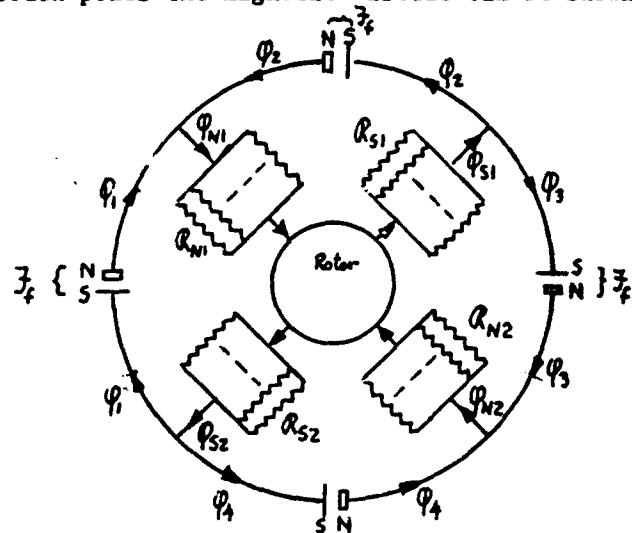


Figure 7: Schematic Diagram Showing the Magnetic Circuit of a Heteropolar Generator

Here, Φ_1 is the flux generated by field coil No. 1, Φ_2 is the flux generated by field coil No. 2, and so on. Φ_1 and Φ_2 combine and make up the flux, Φ_{N1} passing through the first north pole, and this flux returns from the rotor to the stator through the south poles as part of Φ_{S1} and Φ_{S2} .

Similarly, Φ_3 and Φ_4 combine to the flux Φ_{N2} passing through the second north pole, and so on.

The reluctances of the airgaps at the poles are R_{N1} , R_{S1} , R_{N2} and R_{S2} . The mmf's across these airgaps are J_{N1} , J_{S1} , J_{N2} and J_{S2} , respectively.

Since the sum of all mmf's around any closed circuit has to equal zero, Fig. 7 shows that:

$$\bar{J}_{N1} + \bar{J}_{S1} = \bar{J}_{S1} + \bar{J}_{N2} = \bar{J}_{N2} + \bar{J}_{S2} = \bar{J}_{S2} + \bar{J}_{N1} = \bar{J}_f \quad (B.1)$$

from which:

$$\left. \begin{array}{l} \bar{J}_{N1} = \bar{J}_{N2} = \bar{J}_N \\ \bar{J}_{S1} = \bar{J}_{S2} = \bar{J}_S \end{array} \right\} \quad \bar{J}_N + \bar{J}_S = \bar{J}_f \quad (B.2)$$

In general, the generator has n north poles and n south poles, in which case:

$$\left. \begin{array}{l} \bar{J}_{N1} = \bar{J}_{N2} = \dots = \bar{J}_{Nn} = \bar{J}_N \\ \bar{J}_{S1} = \bar{J}_{S2} = \dots = \bar{J}_{Sn} = \bar{J}_S \end{array} \right\} \quad \bar{J}_N + \bar{J}_S = \bar{J}_f \quad (B.3)$$

The flux across the pole airgaps then becomes:

$$\begin{aligned} \varphi_{N1} &= \varphi_1 + \varphi_2 = \frac{\bar{J}_{N1}}{R_{N1}} \\ \varphi_{S1} &= \varphi_2 + \varphi_3 = \frac{\bar{J}_{S1}}{R_{S1}} \\ \varphi_{Nn} &= \varphi_{2n-1} + \varphi_{2n} = \frac{\bar{J}_{Nn}}{R_{Nn}} \\ \varphi_{Sn} &= \varphi_{2n} + \varphi_1 = \frac{\bar{J}_{Sn}}{R_{Sn}} \end{aligned} \quad (B.4)$$

The total flux, φ , is given by:

$$\varphi = \sum_{k=1}^{2n} \varphi_k$$

Thus, from eq. (B.4):

$$\varphi = \left(\frac{1}{R_{N1}} + \frac{1}{R_{N2}} + \dots + \frac{1}{R_{Nn}} \right) \bar{J}_N = \left(\frac{1}{R_{S1}} + \frac{1}{R_{S2}} + \dots + \frac{1}{R_{Sn}} \right) \bar{J}_S \quad (B.5)$$

Set:

$$\frac{1}{R_N} = \sum_{k=1}^n \frac{1}{R_{Nk}} \quad (B.6)$$

$$\frac{1}{R_S} = \sum_{k=1}^n \frac{1}{R_{Sk}}$$

Then:

$$\begin{aligned} \bar{J}_N &= R_N \varphi \\ \bar{J}_S &= R_S \varphi \end{aligned} \quad (B.7)$$

which by means of eq. (B.3) yields:

$$\bar{J}_f = \bar{J}_N + \bar{J}_S = (R_N + R_S)\phi \quad (B.8)$$

or:

$$\begin{aligned} \bar{J}_N &= \frac{R_N}{R_N + R_S} \bar{J}_f \\ \bar{J}_S &= \frac{R_S}{R_N + R_S} \bar{J}_f \end{aligned} \quad (B.9)$$

To determine the reluctances, consider the airgaps at a pole:

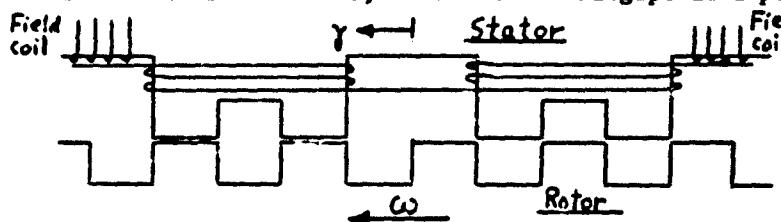


Figure 8: Pole Airgaps At Time t=0

Let there be n_r rotor teeth in total. Then a rotor tooth or a stator tooth extends over an angle $\frac{\pi}{n_r}$. Thus, if the angle γ measured from the centerline of the pole, is fixed in the stator, the position of the centers of the stator teeth are:

$$\gamma_j = \pm \frac{3\pi}{2n_r}, \pm \frac{7\pi}{2n_r}, \dots, \pm \frac{(4j-1)\pi}{2n_r} \quad (B.10)$$

when there are $2n_s$ stator teeth per pole. Note:

$$n_r = 2n(2n_s+1) \quad (B.11)$$

Introduce an x-axis which passes between the last south pole (number n) and the first north pole. Measured from this axis the centerline of the k'th pole is located at:

$$\frac{\pi}{2n} + \frac{\pi}{n}(k-1) \quad k=1, 2, \dots, 2n \quad (B.12)$$

Thus, the j'th stator tooth at the k'th pole is located an angle: $\frac{\pi}{2n} + \frac{\pi}{n}(k-1) + \gamma_j$ from the x-axis. Assume the rotor to be eccentric such that the center of the rotor is a distance $C\epsilon$ from the stator center, and the angle α

is the angle between the x-axis and the direction of eccentricity. Hence:

$$x = C \epsilon \cos \alpha \quad (B.13)$$

$$y = C \epsilon \sin \alpha$$

where C is the radial clearance for the concentric rotor and ϵ is the eccentricity ratio. Thus, the airgap h_{kj} at the j 'th stator tooth for the k 'th pole becomes:

$$h_{kj} = C [1 - \epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha)] \quad (B.14)$$

Next, let figure 8 apply to the time $t=0$. When the angular speed of the rotor is ω , the flux area of a stator tooth becomes:

$$A_j = \frac{1}{2} A_T (1 \pm \cos(n_r \omega t)) = \frac{1}{2} A_T (1 \pm \cos(\nu t)) \quad (B.15)$$

$$\text{Electrical frequency: } \nu = n_r \omega \quad (B.16)$$

where the plus sign applies to the teeth where γ_j is positive, and the minus sign where γ_j is negative. A_T is the actual area of a stator tooth.

From eqs. (B.14) and (B.15) the reluctance R_{kj} of the airgap at the j 'th stator tooth of the k 'th pole can be expressed as:

$$\frac{1}{R_{kj}} = \frac{\mu A_j}{h_{kj}} = \frac{\mu A_T}{2C} (1 \pm \cos(\nu t)) [1 + \epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha)] \quad (B.17)$$

since $\epsilon \ll 1$. Let the total reluctance for positive γ_j be R_{ak} and for negative γ_j be R_{bk} . Then:

$$\begin{aligned} \frac{1}{R_{ak}} &= \sum_{j=1}^{n_s} \left(\frac{1}{R_{kj}} \right)_{\gamma_j > 0} = \frac{\mu A_T}{2C} \sum_{j=1}^{n_s} (1 + \cos(\nu t)) [1 + \epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + (4j-1)\frac{\pi}{2n_r} - \alpha)] \\ &= \frac{\mu A_T}{2C} (1 + \cos(\nu t)) [n_s + \epsilon (G \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha) - H \sin(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha))] \end{aligned} \quad (B.18)$$

$$\begin{aligned} \frac{1}{R_{bk}} &= \sum_{j=1}^{n_s} \left(\frac{1}{R_{kj}} \right)_{\gamma_j < 0} = \frac{\mu A_T}{2C} \sum_{j=1}^{n_s} (1 - \cos(\nu t)) [1 + \epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - (4j-1)\frac{\pi}{2n_r} - \alpha)] \\ &= \frac{\mu A_T}{2C} (1 - \cos(\nu t)) [n_s + \epsilon (G \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha) + H \sin(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha))] \end{aligned} \quad (B.19)$$

where:

$$G = \sum_{j=1}^n \cos(4j-1) \frac{\pi}{2n_r} \quad (B.20)$$

$$H = \sum_{j=1}^n \sin(4j-1) \frac{\pi}{2n_r} \quad (B.21)$$

The total reluctance, R_k , of the airgaps at the k'th pole is then:

$$\frac{1}{R_k} = \frac{1}{R_{nk}} + \frac{1}{R_{bk}} = \frac{\mu A_T}{C} \left[n_s + \epsilon G \cos\left(\frac{\pi}{n}(k-1) + \frac{T}{2n} - \alpha\right) - \epsilon H \cos(\gamma t) \sin\left(\frac{\pi}{n}(k-1) + \frac{T}{2n} - \alpha\right) \right] \quad (B.22)$$

The first north pole is at $k=1$, the second at $k=3$, and the last at $k=2n-1$. Thus, substitution of eq. (B.22) into eq. (B.6) yields:

$$\frac{1}{R_N} = \sum_{k=1,3,\dots,(2n-1)} \frac{1}{R_k} = \frac{\mu A_T}{C} nn_s \quad (B.23)$$

where the following relationships have been employed:

$$\sum_{k=1,3,\dots,(2n-1)} \cos\left(\frac{\pi}{n}(k-1)\right) = \sum_{k=1}^n \cos\left(k \frac{2\pi}{n}\right) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n \geq 2 \end{cases} \quad (B.24)$$

$$\sum_{k=1,3,\dots,(2n-1)} \sin\left(\frac{\pi}{n}(k-1)\right) = \sum_{k=1}^n \sin\left(k \frac{2\pi}{n}\right) = 0 \quad (B.25)$$

and the case of $n=1$ has been ignored as being of no interest. The first south pole is at $k=2$, the second at $k=4$ and the last at $k=2n$. Then, from eq. (B.22) and (B.6):

$$\frac{1}{R_S} = \sum_{k=2,4,\dots,2n} \frac{1}{R_k} = \frac{\mu A_T}{C} nn_s \quad (B.26)$$

Therefore, $R_S = R_N$ and eq. (B.9) yields:

$$\bar{J}_N = \bar{J}_S = \frac{1}{2} \bar{J}_s \quad (B.27)$$

Having established the mmf across the pole airgaps, the flux density B_{kj} at the j 'th stator tooth of the k 'th pole becomes:

$$B_{kj} = \frac{\frac{t}{2} \mathcal{I}_f}{A_j R_{kj}} = \frac{\mu \mathcal{I}_f}{2 h_{kj}} = B_0 [1 + \epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha)] \quad (B.28)$$

where:

$$B_0 = \frac{\mu \mathcal{I}_f}{2C} \quad (B.29)$$

B_0 is the average flux density. The force acting on the stator tooth has an x -component and a y -component which are determined by:

$$\begin{cases} (F_x)_{kj} \\ (F_y)_{kj} \end{cases} = \bar{Q} A_j B_{kj} \begin{cases} \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) \\ \sin(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) \end{cases} \quad (B.30)$$

where, from eq. (B.28):

$$\begin{aligned} B_{kj} &= B_0^2 [1 + 2\epsilon \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j - \alpha)] \\ &= \frac{1}{C} B_0^2 [C + 2x \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) + 2y \sin(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j)] \end{aligned} \quad (B.31)$$

x and y are the rotor displacements from eq. (B.13). Substituting eq. (B.31) into eq. (B.30) there will appear the following products:

$$\begin{aligned} \cos^2(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) &= \frac{1}{2} [1 + \cos 2(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j)] \\ \sin^2(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) &= \frac{1}{2} [1 - \cos 2(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j)] \\ \cos(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) \sin(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) &= \frac{1}{2} \sin 2(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} + \gamma_j) \end{aligned} \quad (B.32)$$

Now, the total magnetic force components are given by:

$$F_x = \sum_{k=1}^{2n} \sum_{j=1}^{n_2} (F_x)_{kj} \quad F_y = \sum_{k=1}^{2n} \sum_{j=1}^{n_2} (F_y)_{kj} \quad (B.33)$$

The following relationships hold true:

$$\sum_{k=1}^{2n} \cos\left(\frac{2\pi}{n}(k-1)\right) = \begin{cases} 2 & \text{for } n=1 \\ 0 & \text{for } n \geq 2 \end{cases} \quad (B.34)$$

$$\sum_{k=1}^{2n} \sin\left(\frac{2\pi}{n}(k-1)\right) = 0 \quad (B.35)$$

If these relationships are used together with the similar ones of eqs. (B.24) and (B.25), substitution of eq. (B.31) into (B.30) and summing according to eq. (B.33) yields:

$$F_x = \bar{Q} \frac{A_T B_o^2}{C} \frac{1}{2} [1 + \cos(yt) + 1 - \cos(yt)] 2nn_s x = \bar{Q} \frac{2nn_s A_T B_o^2}{C} x \quad (B.36)$$

$$F_y = \bar{Q} \frac{2nn_s A_T B_o^2}{C} y \quad (B.37)$$

It is seen that the forces are purely static and are not dependent on time. Thus, the magnetic forces for a heteropolar generator with no load do not cause the rotor to whirl. However, they do contribute a negative stiffness:

$$Q_0 = \bar{Q} \frac{2nn_s A_T B_o^2}{C} \quad (B.38)$$

which must be taken into account if the unbalance response of the rotor is calculated or the hydrodynamic whirl instability is being checked. Of course, if Q_0 exceeds the combined bearing stiffness, the rotor is statically unstable.

APPENDIX III:Magnetic Forces of a Two-Coil Lundell Generator

A two-coil Lundell generator is shown schematically in Figure 3. There are two field coils in this generator, one on each side of the central plane which contains the northpoles. The magnetic flux path goes from the northpoles of the rotor (Fig. 3a) through the stator to the southpoles of the rotor (see Fig. 9), and then through the cylindrical air gaps g_1 and g_3 (Fig. 3b), the stationary pieces on which the field coils are wound, the conical airgaps g_2 and g_4 and back to the northpoles of the rotor. The magnetic circuit is shown diagrammatically in Fig. 10. The reluctances of the airgaps g_1 , g_2 , g_3 and g_4 are respectively represented by R_1 , R_2 , R_3 and R_4 . Each field winding produces an m.m.f. of \mathcal{F}_f . Let there be n north poles and n south poles, and let R_N and R_S be the total reluctances of the airgaps of the north poles and the south poles, respectively. From the magnetic circuit shown in Fig. 10 we have:

$$\mathcal{F}_f = \Phi (R_N + R_S) + \Phi_1 (R_1 + R_2) \quad (C.1)$$

$$\Phi_1 (R_1 + R_2) = \Phi_3 (R_3 + R_4) \quad (C.2)$$

$$\Phi = \Phi_1 + \Phi_3 \quad (C.3)$$

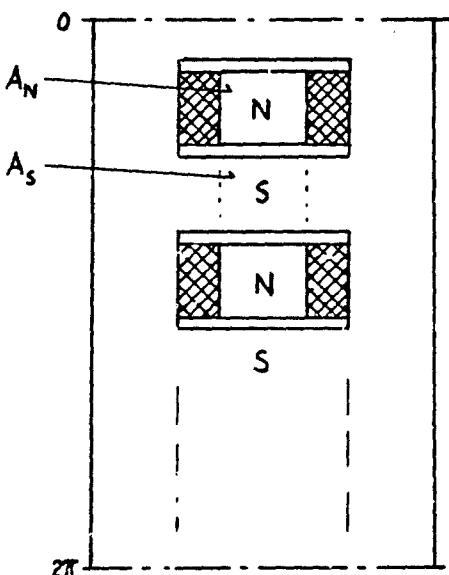


Figure 9
Expanded view of outer surface of rotor.

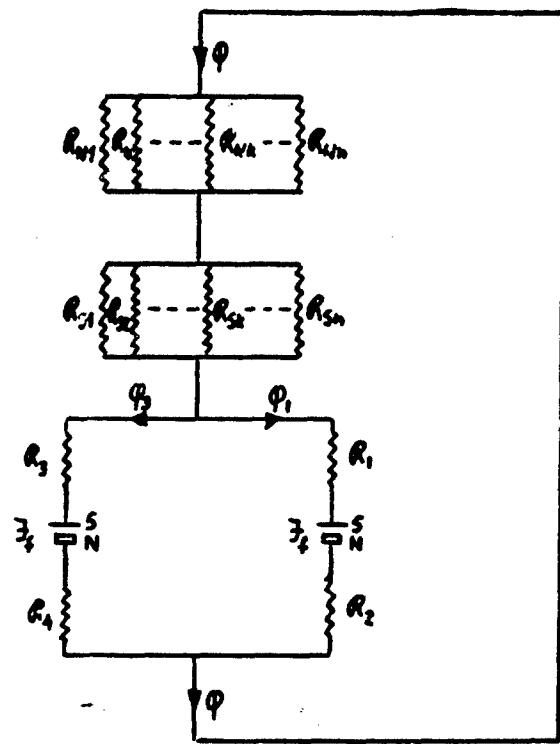


Figure 10
Magnetic Circuit for a Two-Coil Lundell Generator

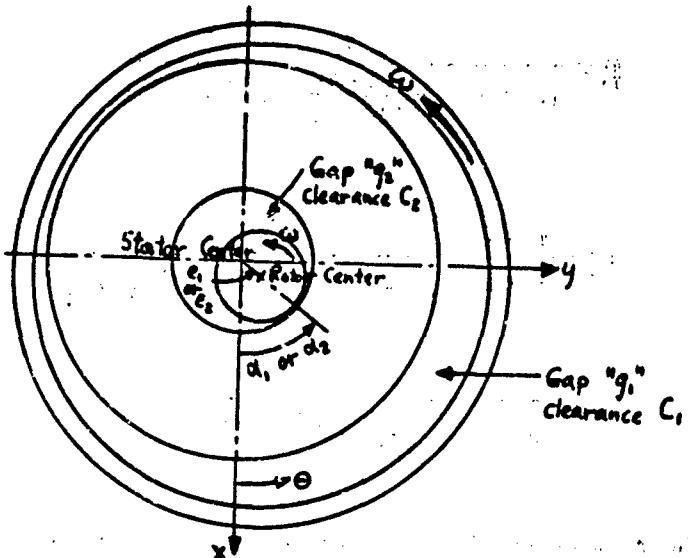


Figure 11
Section "R-R" of Fig. 3b

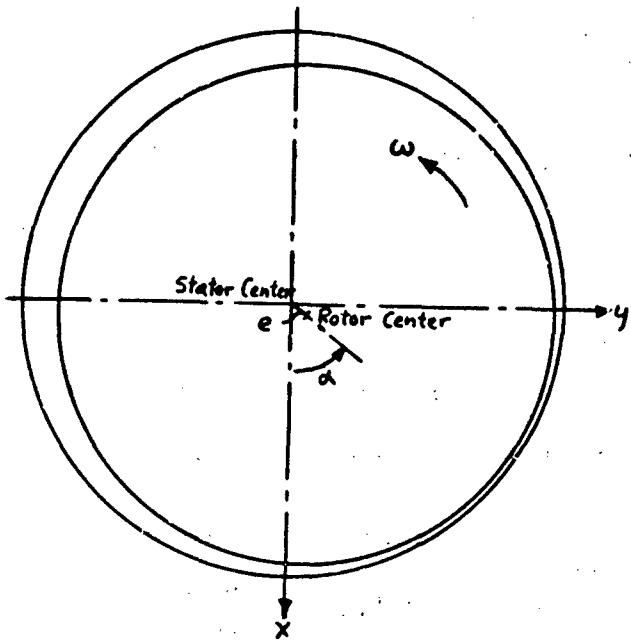


Figure 12
Section "C-C" of Fig. 3b

Solving the above three equations for Φ_1 , Φ_3 and Φ , we obtain:

$$\Phi_1 = \frac{\bar{J}_f}{(R_1+R_2) + \left(\frac{R_1+R_2}{R_3+R_4} + 1\right)(R_N+R_S)} \quad (C.4)$$

$$\Phi_3 = \frac{R_1+R_2}{R_3+R_4} \cdot \frac{\bar{J}_f}{(R_1+R_2) + \left(\frac{R_1+R_2}{R_3+R_4} + 1\right)(R_N+R_S)} \quad (C.5)$$

$$\Phi = \frac{\bar{J}_f}{(R_1+R_2)\left(\frac{R_1+R_2}{R_3+R_4} + 1\right)^{-1} + (R_N+R_S)} \quad (C.6)$$

To calculate the reluctance, R_1 , of the airgap g_1 , let us consider the airgap as infinitesimally many reluctances connected in parallel. Thus, referring to Figure 11, we have:

$$\frac{1}{R_1} = \int_0^{2\pi} \frac{\mu L_1 r d\theta}{h_1} \quad (C.7)$$

where: h_1 = film thickness of airgap $g_1 = C_1 [1 + \epsilon_1 \cos(\theta - \alpha_1)]$

$$\epsilon_1 = \frac{e_1}{C_1} \quad (C.8)$$

L_1 = axial length of airgap g_1

e_1 and C_1 are the eccentricity and the mean film thickness of airgap g_1 .
 r is the mean radius of the cylindrical airgap

Substituting eq. (C.8) into (C.7) and neglecting terms of the order ϵ_1^2 :

$$\frac{1}{R_1} = \frac{\mu L_1 r}{C_1} \int_0^{2\pi} [1 - \epsilon_1 \cos(\theta - \alpha_1)] d\theta = \frac{2\pi \mu L_1 r}{C_1}$$

or:

$$R_1 = \frac{C_1}{\mu L_1 2\pi r} \quad (C.9)$$

Similarly, the airgap g_3 has the reluctance:

$$R_3 = \frac{C_3}{\mu L_3 2\pi r} \quad (C.10)$$

and for the conical airgaps g_2 and g_4 ,

$$R_2 = \frac{C_2}{\mu L_2 2\pi \bar{r}} \quad (C.11)$$

$$R_4 = \frac{C_4}{\mu L_4 2\pi \bar{r}} \quad (C.12)$$

where \bar{r} = arithmetic mean radius. In general, because of manufacturing tolerances:

$$\begin{aligned} C_1 &\neq C_3 \\ C_2 &\neq C_4 \end{aligned} \quad (C.13)$$

Let R_{Nk} be the reluctance of the airgap at the k 'th north pole:

$$R_{Nk} = \frac{C}{\mu A_N} [1 - \epsilon \cos(\theta_{Nk} + \omega t - \alpha)] \quad (C.14)$$

Similarly,

$$R_{Sk} = \frac{C}{\mu A_S} [1 - \epsilon \cos(\theta_{Sk} + \omega t - \alpha)] \quad (C.15)$$

where:

$$\begin{aligned} \theta_{Nk} &= \frac{2\pi}{n}(k-1) \\ \theta_{Sk} &= \frac{2\pi}{n}(k-1) + \frac{\pi}{n} \end{aligned} \quad \epsilon = \frac{e}{C} \quad (C.16)$$

Now:

$$\frac{1}{R_N} = \sum_{k=1}^n \frac{1}{R_{Nk}} = \frac{\mu A_N}{C} \sum_{k=1}^n [1 + \epsilon \cos(\theta_{Nk} + \omega t - \alpha)] = \begin{cases} \frac{\mu A_N}{C} [1 + \epsilon \cos(\omega t - \alpha)] & \text{for } n=1 \\ n \frac{\mu A_N}{C} & \text{for } n \geq 2 \end{cases} \quad (C.17)$$

By the same procedure:

$$\frac{1}{R_S} = \begin{cases} \frac{\mu A_S}{C} [1 + \epsilon \cos(\omega t + \pi - \alpha)] & \text{for } n=1 \\ n \frac{\mu A_S}{C} & \text{for } n \geq 2 \end{cases} \quad (C.18)$$

From here on, we assume that the generator has at least two pairs of poles.
Thus for $n \geq 2$:

$$\begin{aligned} R_N &= \frac{C}{\mu n A_N} \\ R_S &= \frac{C}{\mu n A_S} \end{aligned} \quad (C.19)$$

So far, we have obtained R_1, R_2, R_3 and R_4 (Eqs. (C.9) to (C.12)), and R_N and R_S (Eq. (C.19)). Hence for a given field m.m.f. $\bar{\Phi}$, we can calculate φ_1, φ_3 and φ from Eqs. (C.4) to (C.6). The magnetic flux through the individual north and southpoles can be expressed by:

$$\varphi_{Nk} = \frac{R_N}{R_{Nk}} \varphi \quad (C.20)$$

$$\varphi_{Sk} = \frac{R_S}{R_{Sk}} \varphi \quad (C.21)$$

Using eqs. (C.14) and (C.15), and for small ϵ , we obtain:

$$\varphi_{Nk} = \frac{\varphi}{n} [1 + \epsilon \cos(\theta_{Nk} + \omega t - \alpha)] \quad n \geq 2 \quad (C.22)$$

$$\varphi_{Sk} = \frac{\varphi}{n} [1 + \epsilon \cos(\theta_{Sk} + \omega t - \alpha)] \quad n \geq 2 \quad (C.23)$$

The x and y components of the magnetic force are:

$$F_{Nx} = \frac{\bar{Q}}{A_N} \sum_{k=1}^n \varphi_{Nk}^2 \cos(\theta_{Nk} + \omega t) = \frac{\bar{Q}}{A_N} \left(\frac{\varphi}{n} \right)^2 \sum_{k=1}^n [1 + 2\epsilon \cos(\theta_{Nk} + \omega t - \alpha)] \cos(\theta_{Nk} + \omega t) \quad (C.24)$$

$$F_{Ny} = \frac{\bar{Q}}{A_S} \left(\frac{\varphi}{n} \right)^2 \sum_{k=1}^n [1 + 2\epsilon \cos(\theta_{Nk} + \omega t - \alpha)] \sin(\theta_{Nk} + \omega t) \quad (C.25)$$

Using the relationships:

$$\sum_{k=1}^n \cos\left(\frac{2\pi}{n}(k-1)\right) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n \geq 2 \end{cases} \quad (\text{C.26})$$

$$\sum_{k=1}^n \sin\left(\frac{2\pi}{n}(k-1)\right) = 0 \quad (\text{C.26})$$

$$\sum_{k=1}^n \cos\left(\frac{4\pi}{n}(k-1)\right) = \begin{cases} 1 & \text{for } n=1 \\ 2 & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases} \quad (\text{C.27})$$

$$\sum_{k=1}^n \sin\left(\frac{4\pi}{n}(k-1)\right) = 0 \quad (\text{C.27})$$

eq. (C.24) is readily reduced to

$$\begin{aligned} F_{Nx} &= \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{n}\right)^2 \sum_{k=1}^n 2\varepsilon \cos(\theta_{Nk} + \omega t - \alpha) \cos(\theta_{Nk} + \omega t) \\ &= \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{n}\right)^2 \varepsilon \sum_{k=1}^n \{ \cos(2\theta_{Nk} + 2\omega t - \alpha) + \cos \alpha \} \\ &= \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{n}\right)^2 \varepsilon \sum_{k=1}^n \{ \cos \alpha + (\cos \alpha \cdot \cos(2\omega t) + \sin \alpha \cdot \sin(2\omega t)) \cos(2\theta_{Nk}) \} \end{aligned}$$

Thus,

$$F_{Nx} = \begin{cases} \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{2}\right)^2 2\varepsilon [\cos \alpha (1 + \cos(2\omega t)) + \sin \alpha \cdot \sin(2\omega t)] & \text{for } n=2 \\ n \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{n}\right)^2 \varepsilon \cos \alpha & \text{for } n \geq 3 \end{cases} \quad (\text{C.28})$$

Similarly,

$$F_{Ny} = \begin{cases} \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{2}\right)^2 2\varepsilon [\cos \alpha \cdot \sin(2\omega t) + \sin \alpha (1 - \cos(2\omega t))] & \text{for } n=2 \\ n \frac{\bar{Q}}{A_N} \left(\frac{\Phi}{n}\right)^2 \varepsilon \sin \alpha & \text{for } n \geq 3 \end{cases} \quad (\text{C.29})$$

$$F_{Sx} = \begin{cases} \frac{\bar{Q}}{A_s} \left(\frac{\Phi}{2}\right)^2 2\varepsilon [\cos \alpha (1 - \cos(2\omega t)) - \sin \alpha \cdot \sin(2\omega t)] & \text{for } n=2 \\ n \frac{\bar{Q}}{A_s} \left(\frac{\Phi}{n}\right)^2 \varepsilon \cos \alpha & \text{for } n \geq 3 \end{cases} \quad (\text{C.30})$$

$$F_{Sy} = \begin{cases} \frac{\bar{Q}}{A_s} \left(\frac{\Phi}{2}\right)^2 2\varepsilon [-\cos \alpha \sin(2\omega t) + \sin \alpha (1 + \cos(2\omega t))] & \text{for } n=2 \\ n \frac{\bar{Q}}{A_s} \left(\frac{\Phi}{n}\right)^2 \varepsilon \sin \alpha & \text{for } n \geq 3 \end{cases} \quad (\text{C.31})$$

Thus, it is seen that for generators with at least three pairs of poles, the magnetic forces due to the north and south poles are time-independent. For $n = 2$, the magnetic forces are functions of time as indicated by the above equations, if $A_N \neq A_S$. If, however, $A_N = A_S$ and ($n=2$), then the time-dependent parts of the north and southpoles cancel with each other, and the resultant ($F_{Nx} + F_{Sx}$) and ($F_{Ny} + F_{Sy}$) are again time-independent.

If the displacements of the rotor center are x and y , it is seen from Fig. 12 and so on that:

$$x = e \cos \alpha = C \varepsilon \cos \alpha$$

$$y = e \sin \alpha = C \varepsilon \sin \alpha$$

Furthermore, introduce the flux densities:

$$B_N = \frac{\Phi}{n A_N}$$

$$B_S = \frac{\Phi}{n A_S}$$

Then eqs. (C.28) to (C.31) can be written:

$$F_{Nx} = \begin{cases} 2 \bar{Q} \frac{A_N B_N^2}{C} [x(1+\cos(2\omega t)) + y \sin(2\omega t)] & \text{for } n=2 \\ n \bar{Q} \frac{A_N B_N^2}{C} x & \text{for } n=3 \end{cases} \quad (C.32)$$

$$F_{Ny} = \begin{cases} 2 \bar{Q} \frac{A_N B_N^2}{C} [x \sin(2\omega t) + y(1-\cos(2\omega t))] & \text{for } n=2 \\ n \bar{Q} \frac{A_N B_N^2}{C} y & \text{for } n=3 \end{cases} \quad (C.33)$$

$$F_{Sx} = \begin{cases} 2 \bar{Q} \frac{A_S B_S^2}{C} [x(1-\cos(2\omega t)) - y \sin(2\omega t)] & \text{for } n=2 \\ n \bar{Q} \frac{A_S B_S^2}{C} x & \text{for } n=3 \end{cases} \quad (C.34)$$

$$F_{Sy} = \begin{cases} 2 \bar{Q} \frac{A_S B_S^2}{C} [-x \sin(2\omega t) + y(1+\cos(2\omega t))] & \text{for } n=2 \\ n \bar{Q} \frac{A_S B_S^2}{C} y & \text{for } n=3 \end{cases} \quad (C.35)$$

These results are identical to the results obtained for the homopolar generator in Appendix I where it is shown how they are used in the stability and the response calculations.

Magnetic Forces at Cylindrical Airgap g_1

The total magnetic flux through the airgap g_1 is Φ_1 , (see Fig. 10) and the total reluctance is R_1 ; they are respectively given by eqs. (C.4) and (C.9). If we use the concept of permeance which is the inverse of reluctance, then:

$$P_1 = \frac{1}{R_1} = \frac{\mu L_1 2\pi r}{C_1} \quad (C.36)$$

For a differential element $r d\theta$, the permeance is:

$$dP_1 = \frac{\mu L_1 r d\theta}{C_1 [1 - \epsilon_1 \cos(\theta - \alpha_1)]} \quad (C.37)$$

Let the flux passing through $r d\theta$ be $d\Phi_1$. Then, from the magnetic circuit:

$$\frac{d\Phi_1}{dP_1} = \frac{\Phi_1}{P_1} = \frac{\Phi_1 C_1}{\mu L_1 2\pi r}$$

or

$$d\Phi_1 = \frac{1 + \epsilon_1 \cos(\theta - \alpha_1)}{2\pi} \Phi_1 d\theta \quad (C.38)$$

Let B_1 be the flux density:

$$B_1 = \frac{d\Phi_1}{L_1 r d\theta} = \frac{1 + \epsilon_1 \cos(\theta - \alpha_1)}{L_1 2\pi r} \quad (C.39)$$

Thus, the x-component of the magnetic forces is:

$$\begin{aligned} F_{ix} &= \bar{Q} \int_0^{2\pi} B_1^2 \cos \theta L_1 r d\theta = \bar{Q} \frac{\Phi_1^2}{L_1 r (2\pi)^2} \int_0^{2\pi} [1 + 2\epsilon_1 \cos(\theta - \alpha_1)] \cos \theta d\theta \\ &= \bar{Q} \frac{\epsilon_1 \Phi_1^2}{L_1 2\pi r} \cos \alpha_1 = \bar{Q} \frac{\Phi_1^2}{2\pi r L_1 C_1} X_1 \end{aligned} \quad (C.40)$$

Similarly, the y-component is:

$$F_{ly} = \bar{Q} \int_0^{2\pi} B_i^2 \sin \theta L_i r d\theta = \bar{Q} \frac{\epsilon_i \Phi_i^2}{L_i 2\pi r} \sin \alpha_i = \bar{Q} \frac{\Phi_i^2}{2\pi r L_i C_i} y_i \quad (C.41)$$

where

$$\begin{aligned} x_i &= e_i \cos \alpha_i = C_i E_i \cos \alpha_i \\ y_i &= e_i \sin \alpha_i = C_i E_i \sin \alpha_i \end{aligned} \quad (C.42)$$

Magnetic Forces at Conical Airgap g_2

The geometry of the conical airgap is shown in the diagram below.

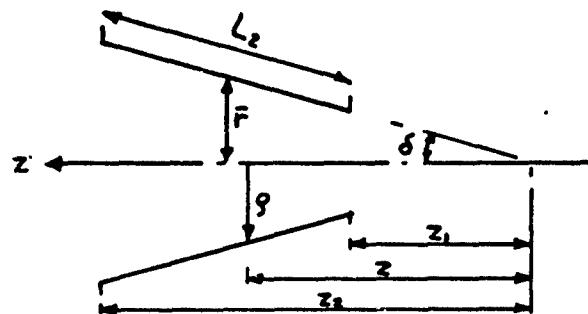


Figure 13: Geometry of Conical Airgap

$$g = z \cdot \tan \delta \quad (C.43)$$

for a small element ($g d\theta \frac{dz}{\cos \delta}$), the permeance is:

$$dP_2 = \frac{\mu_0 d\theta dz / \cos \delta}{C_2 [1 - \epsilon_2 \cos(\theta - \alpha_2)]} \quad (C.44)$$

The subscript "2" is for airgap s_2 . From the magnetic circuit, the flux through the small element is

$$\frac{d\Phi_2}{dP_2} = \frac{\Phi_2}{P_2} \quad (C.45)$$

where $\Phi_2 = \Phi_1$ (see Fig. 10)

and $P_2 = \frac{1}{R_2}$ (use eq. (C.11))

Thus

$$d\Phi_2 = \Phi_1 \frac{C_2}{\mu L_2 2\pi F} \cdot \frac{\mu g d\theta dz / \cos\delta}{C_2} [1 + \epsilon_2 \cos(\theta - \alpha_2)]$$

$$B_2 = \text{flux density} = \frac{d\Phi_2}{g d\theta dz / \cos\delta} = \frac{\Phi_1}{L_2 2\pi F} [1 + \epsilon_2 \cos(\theta - \alpha_2)] \quad (C.46)$$

The corresponding x-component of the magnetic force becomes:

$$F_{2x} = \bar{Q} \int_0^{2\pi} \int_{z_1}^{z_2} B_2^2 \cos\theta g d\theta \frac{dz}{\cos\delta} \cos\delta = \bar{Q} \int_0^{2\pi} \int_{z_1}^{z_2} \left(\frac{\Phi_1}{L_2 2\pi F} \right)^2 [1 + 2\epsilon_2 \cos(\theta - \alpha_2)] \cos\theta d\theta z \tan\delta dz$$

$$= \bar{Q} \left(\frac{\Phi_1}{L_2 2\pi F} \right)^2 2\epsilon_2 \pi \cos\alpha_2 \tan\delta \int_{z_1}^{z_2} z dz \quad (C.47)$$

But: $\tan\delta \int_{z_1}^{z_2} z dz = (\tan\delta) \frac{1}{2} (z_2 + z_1)(z_2 - z_1) = (\tan\delta) \frac{1}{2} (z_2 + z_1) L_2 \cos\delta = F L_2 \cos\delta$

i.e.:

$$F_{2x} = \bar{Q} \frac{\Phi_1^2}{L_2 2\pi F} \epsilon_2 \cos\delta \cos\alpha_2 = \bar{Q} \frac{\Phi_1^2 \cos\delta}{2\pi F L_2 C_2} x_2 \quad (C.48)$$

Similarly,

$$F_{2y} = \bar{Q} \frac{\Phi_1^2}{L_2 2\pi F} \epsilon_2 \cos\delta \sin\alpha_2 = \bar{Q} \frac{\Phi_1^2 \cos\delta}{2\pi F L_2 C_2} y_2 \quad (C.49)$$

where:

$$x_2 = C_2 \epsilon_2 \cos\alpha_2$$

$$y_2 = C_2 \epsilon_2 \sin\alpha_2 \quad (C.50)$$

For gap "3" and gap "4", we have by the same procedure,

$$F_{3x} = \bar{Q} \frac{\phi_3^2}{L_1 2\pi r} \epsilon_3 \cos \alpha_3 = \bar{Q} \frac{\phi_3^2}{2\pi r L_1 C_3} x_3 \quad (C.51)$$

$$F_{3y} = \bar{Q} \frac{\phi_3^2}{L_1 2\pi r} \epsilon_3 \sin \alpha_3 = \bar{Q} \frac{\phi_3^2}{2\pi r L_1 C_3} y_3 \quad (C.52)$$

$$F_{4x} = \bar{Q} \frac{\phi_3^2}{L_2 2\pi F} \cos \delta \epsilon_4 \cos \alpha_4 = \bar{Q} \frac{\phi_3^2 \cos \delta}{2\pi F L_2 C_4} x_4 \quad (C.53)$$

$$F_{4y} = \bar{Q} \frac{\phi_3^2}{L_2 2\pi F} \cos \delta \epsilon_4 \sin \alpha_4 = \bar{Q} \frac{\phi_3^2 \cos \delta}{2\pi F L_2 C_4} y_4 \quad (C.54)$$

Thus, the forces F_{1x} to F_{4y} can be represented by simple negative springs in the rotor response and rotor stability calculations.

APPENDIX IV: FIELD DUE TO ARMATURE REACTION OF A THREE-PHASE WINDING

In this appendix the magnetic field produced by the armature reaction will be studied. Consider Fig. 14 where the poles of the rotor move to the right:

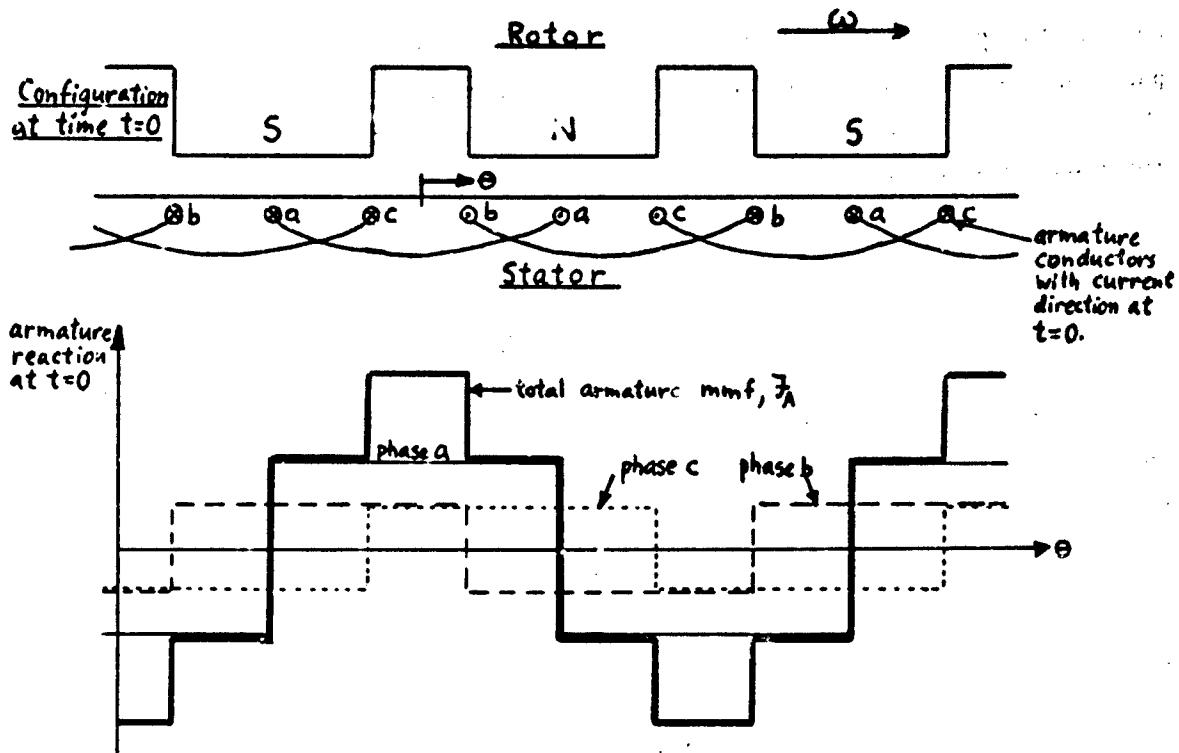


Figure 14: Armature Current Directions and Instantaneous mmf at $t = 0$. This figure shows the rotor-stator position, the armature current directions and the instantaneous armature mmf at time $t = 0$. The convention for current directions are: \otimes means "into the paper" and \odot means "out of the paper". To arrive at Figure D.1, assume that the airgaps at all the poles are the same. With $2n$ poles (n north poles and n south poles), the pole spacing becomes $\frac{\pi}{n}$ whereby the flux passing through the armature coils is:

$$\text{for coil "a"} \quad \Phi_a = i\varphi e^{in\omega t} \quad (\text{D.1})$$

$$\text{for coil "b"} \quad \Phi_b = i\varphi e^{i(n\omega t - \frac{\pi}{3})} \quad (\text{D.2})$$

$$\text{for coil "c"} \quad \Phi_c = i\varphi e^{i(n\omega t - \frac{2\pi}{3})} \quad (\text{D.3})$$

where:

$$i = \sqrt{-1}$$

and all the Φ 's are complex numbers. The convention is that only the real part applies. To illustrate, assume that the flux Φ is generated solely by the field coils. Then Φ is real (i.e. there is no phase shift between the rotor motion and the flux) and eqs. (D.1) to (D.3) can be written as:

$$\begin{aligned}\Phi_a &= \operatorname{Re}\{i\Phi e^{in\omega t}\} = \Phi \operatorname{Re}\{i(\cos(n\omega t) + i\sin(n\omega t))\} = -\Phi \sin(n\omega t), \\ \Phi_b &= \operatorname{Re}\{i\Phi e^{i(n\omega t - \frac{\pi}{3})}\} = -\Phi \sin(n\omega t - \frac{\pi}{3}) \\ \Phi_c &= \operatorname{Re}\{i\Phi e^{i(n\omega t - \frac{2\pi}{3})}\} = -\Phi \sin(n\omega t - \frac{2\pi}{3})\end{aligned}\quad (\text{D.4})$$

Under load Φ will lag the rotor motion as discussed later in which case a phase angle is introduced in eqs. (D.4).

Let each armature coil have N_A turns. The the induced voltage is

$$\text{for coil "a"} \quad e_a = -N_A \frac{d\Phi}{dt} = n\omega N_A \Phi \quad (\text{D.5})$$

$$\text{for coil "b"} \quad e_b = n\omega N_A \Phi e^{-i\frac{\pi}{3}} \quad (\text{D.6})$$

$$\text{for coil "c"} \quad e_c = n\omega N_A \Phi e^{-i\frac{2\pi}{3}} \quad (\text{D.7})$$

where the exponential $e^{in\omega t}$ has been left out for simplification. This will also be done in the following but it must always be recalled that properly this factor belongs in the equations.

The corresponding armature currents are:

$$\text{for coil "a"} \quad i_a = \frac{e_a}{Z_A} = \frac{n\omega N_A}{Z_A} \Phi \quad (\text{D.8})$$

and similarly for the other two coils, where Z_A is the impedance of the output circuit. Setting $t=0$ in the above equations and using the right hand rule, the current directions in the armature conductors come out as shown in Figure 14. The corresponding mmf of the armature coils (the armature reaction) is found as:

$$\text{for coil "a"} \quad \text{mmf}_a = N_A i_a = \frac{n\omega N_A^2}{Z_A} \phi \quad (\text{D.9})$$

and similarly for coils "b" and "c". Hence, at time $t=0$ the armature mmf's are distributed around the circumference of the stator as a function of the angular coordinate θ as shown in Figure 14. If they be represented by their fundamental harmonics instead of the "rectangular waves" shown in Figure 14, the mmf's of the three phases become:

$$\text{for phase "a"} \quad \mathfrak{J}_a = \frac{4}{\pi} \text{mmf}_a \cos(n\theta) \quad (\text{D.10})$$

$$\text{for phase "b"} \quad \mathfrak{J}_b = \frac{4}{\pi} \text{mmf}_b \cos(n\theta - \frac{2\pi}{3}) \quad (\text{D.11})$$

$$\text{for phase "c"} \quad \mathfrak{J}_c = \frac{4}{\pi} \text{mmf}_c \cos(n\theta - \frac{4\pi}{3}) \quad (\text{D.12})$$

where the factor $\frac{4}{\pi}$ derives from taking the first harmonic of a rectangular wave. Here the origin for the angle θ is between two poles at $t=0$ as shown in Figure 14.

To find the total armature mmf, \mathfrak{J}_A , the mmf's of the three phases must be added:

$$\begin{aligned} \mathfrak{J}_A &= \mathfrak{J}_a + \mathfrak{J}_b + \mathfrak{J}_c = \frac{4}{\pi} \frac{n\omega N_A^2}{Z_A} \phi \left\{ \cos(n\theta) + e^{-i\frac{2\pi}{3}} \cos(n\theta - \frac{2\pi}{3}) + e^{-i\frac{4\pi}{3}} \cos(n\theta - \frac{4\pi}{3}) \right\} e^{in\omega t} \\ &= \frac{4}{\pi} \frac{n\omega N_A^2}{Z_A} \phi \cdot \frac{3}{2} [\cos(n\theta) - i\sin(n\theta)] e^{in\omega t} = \frac{6}{\pi} \frac{n\omega N_A^2}{Z_A} \phi e^{i(n\omega t - \theta)} \end{aligned} \quad (\text{D.13})$$

This represents a wave travelling synchronous with the rotor.

Let the load be balanced and let the effective value or ammeter value of the

current per phase be I_A . Then, from eq. (D.8):

$$I_A = \frac{|i_A|}{\sqrt{2}} = \frac{n\omega N_A}{\sqrt{2}} \frac{|\phi|}{|Z_A|} \quad (D.14)$$

Similarly, let the effective value or voltmeter value of the line voltage per phase be E_A so that from eq. (D.5):

$$E_A = \frac{|e_A|}{\sqrt{2}} = \frac{n\omega N_A}{\sqrt{2}} |\phi| \quad (D.15)$$

Introduce the power factor angle ψ :

$$\text{power factor} = \cos \psi = \frac{\operatorname{Re}\{Z_A\}}{|Z_A|} \quad \sin \psi = \frac{\operatorname{Im}\{Z_A\}}{|Z_A|} \quad (D.16)$$

which means:

$$\frac{1}{Z_A} = \frac{1}{|Z_A|} (\cos \psi - i \sin \psi) = \frac{1}{|Z_A|} e^{-i\psi} \quad (D.17)$$

The physical interpretation of the power factor angle ψ can be obtained by substituting eq. (D.17) into eq. (D.8):

$$i_A = \frac{e_A}{Z_A} = \frac{e_A}{|Z_A|} e^{-i\psi} \quad (D.18)$$

or, in other words, ψ gives the phase angle by which the armature current lags the voltage.

Next, introduce the power angle δ by the equations:

$$\cos \delta = \frac{\operatorname{Re}\{\phi\}}{|\phi|} \quad \sin \delta = \frac{-\operatorname{Im}\{\phi\}}{|\phi|} \quad (D.19)$$

or:

$$\phi = |\phi| (\cos \delta - i \sin \delta) = |\phi| e^{-i\delta} \quad (D.20)$$

Hence, δ gives the angle by which the flux lags the rotor rotation. It is called the power angle since it is the angle the rotor must pull ahead of the resultant magnetic field to supply the required load. The angle can be measured actually on the generator by means of a stroboscope. If the physical angle is measured as δ' , then:

$$\delta = n\delta' \quad (D.21)$$

since the electrical frequency is n times greater than the mechanical rotational frequency.

In order to understand the relationship between the armature mmf and the airgap flux Φ , it is useful to consider the phasor diagram of the alternator where each of the above quantities are taken as vectors rotating with the angular speed ($n\omega$). Comparing eqs. (D.1) and (D.5) it is seen that the line voltage lags the flux Φ by 90 degrees. The total mmf, \mathcal{B} , required to produce this flux is, of course, in phase with the flux. The armature reaction, \mathcal{B}_A , is in phase with the armature current (eq. (D.9)) which, as discussed above, lags the line voltage by the power factor angle ψ :

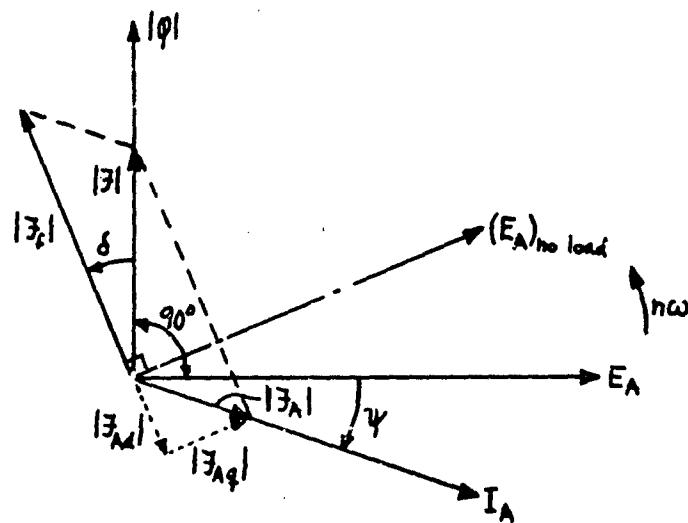


Figure 15: Phasor Diagram

The total mmf, \bar{J} , is the vector sum of the field mmf, \bar{J}_f , and the armature reaction:

$$\bar{J} = \bar{J}_f + \bar{J}_A \quad (D.22)$$

Thus, to offset \bar{J}_A and meet the required total mmf, the field mmf, \bar{J}_f , must lead the flux ϕ . The lead angle is the power angle δ as discussed above.

To express the armature reaction \bar{J}_A , substitute eqs. (D.17) and (D.20) into eq. (D.13):

$$\bar{J}_A = \frac{6}{\pi} n \omega N_A^2 \frac{|\phi|}{|Z_A|} e^{i(n(\omega t - \theta) - \psi - \delta)} \quad (D.23)$$

or by introducing the effective value of the armature current, I_A , from eq. (D.14):

$$\bar{J}_A = \frac{6\sqrt{2}}{\pi} N_A I_A \cos(n(\omega t - \theta) - \psi - \delta) \quad (D.24)$$

In practice, windings are fractionally pitched and distributed to provide better design (efficiency, wave form and winding configuration). This produces a spatial phase angle between the conductors of a given coil (or winding) so that the mmf per phase is reduced slightly from the value it would have for a concentrated, full pitch winding. Hence, eq. (D.24) is modified to:

$$\bar{J}_A = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(n(\omega t - \theta) - \psi - \delta) \quad (D.25)$$

where

$$K_d = \text{distribution factor } (K_d \leq 1) \quad (D.26)$$

$$K_p = \text{pitch factor } (K_p \leq 1) \quad (D.27)$$

$K_d = 1$ for a concentrated winding and may be taken as $K_d \approx 0.9$ for a distributed winding. $K_p = 1$ for a full pitch winding and $K_p = 0.9$ for a 5/6 pitch winding.

The armature reaction can be decomposed into a de-magnetizing component \bar{J}_{Ad} in line with the field mmf, \bar{J}_f , and a cross-magnetizing component \bar{J}_{Aq} lagging \bar{J}_f by 90 degrees. From eq. (D.25):

$$\bar{J}_A = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A [\cos(\psi + \delta) \cos(n(\omega t - \phi)) + \sin(\psi + \delta) \sin(n(\omega t - \phi))] = \bar{J}_{Aq} + \bar{J}_{Ad} \quad (\text{D.28})$$

i.e.

$$\bar{J}_{Aq} = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi + \delta) \cdot \cos(n(\omega t - \phi)) \quad (\text{D.29})$$

$$\bar{J}_{Ad} = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi + \delta) \cdot \sin(n(\omega t - \phi)) \quad (\text{D.30})$$

or:

$$|\bar{J}_{Aq}| = \bar{J}_A \cos(\psi + \delta) = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi + \delta) \quad (\text{D.31})$$

$$|\bar{J}_{Ad}| = \bar{J}_A \sin(\psi + \delta) = \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi + \delta) \quad (\text{D.32})$$

as readily seen from the phasor diagram in Figure D.2.

With these results, the circumferential distribution of mmf can be shown schematically as (see also Figure 14):

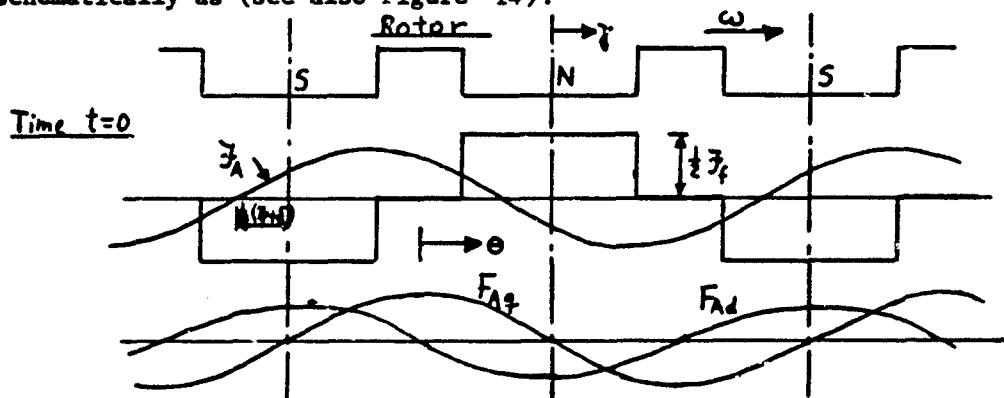


Figure 16: Circumferential mmf Distribution

The total field mmf is distributed evenly among the poles such that the field contributes $\frac{1}{2} \bar{\mathcal{I}}_c$ to each pole airgap (see Appendix I). This field follows the rotor in its rotation. Thus, if instead as in the foregoing where the armature reaction has been expressed as a function of the stationary coordinate θ , it is instead "seen" from the rotor, the angular coordinate γ , fixed in the rotor becomes (see Figure 16):

$$\gamma + \frac{\bar{\mathcal{I}}}{2n} = \theta - \omega t \quad (D.33)$$

such that eqs. (D.29) and (D.30) yield:

$$\bar{\mathcal{I}}_{Aq} = - |\bar{\mathcal{I}}_{Aq}| \sin(\gamma) \quad (D.34)$$

$$\bar{\mathcal{I}}_{Ad} = - |\bar{\mathcal{I}}_{Ad}| \cos(\gamma) \quad (D.35)$$

where $|\bar{\mathcal{I}}_{Aq}|$ and $|\bar{\mathcal{I}}_{Ad}|$ are given by eqs. (D.31) and (D.32). It is seen that the de-magnetizing component, as the name implies, opposes the field mmf except for the unlikely case where $(\gamma + \delta) < 0$.

APPENDIX V: Magnetic Forces of a Homopolar Generator - Modifications due to Generator Load

In Appendix I the magnetic forces of a homopolar generator operating under no load have been derived. In this appendix the effect of load will be investigated.

In Appendix IV it is shown that the mmf across the airgap at any pole is made up of three components: $\frac{1}{2} \bar{B}_f$ (\bar{B}_f is the mmf of the single field coil), the de-magnetizing component \bar{B}_{Ad} and the cross-magnetizing component \bar{B}_{Ag} , where the latter two components make up the armature reaction (see Figure 16 Appendix IV). Thus, with the angular coordinate γ fixed in the rotor and measured from a pole centerline, the airgap mmf at the north poles and at the southpoles can be written:

$$\bar{B}_N = \bar{B}_S = \frac{1}{2} \bar{B}_f [1 - f_A \cos(n\gamma) + f_q \sin(n\gamma)] \quad (E.1)$$

where:

$$f_A = \frac{|\bar{B}_{Ad}|}{\frac{1}{2} \bar{B}_f} = \frac{2}{\bar{B}_f} \cdot \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \sin(\psi + \delta) \quad (E.2)$$

$$f_q = \frac{|\bar{B}_{Ag}|}{\frac{1}{2} \bar{B}_f} = \frac{2}{\bar{B}_f} \cdot \frac{6\sqrt{2}}{\pi} K_d K_p N_A I_A \cos(\psi + \delta) \quad (E.3)$$

(see eqs. (D.31) and (D.32), and eqs. (D.34) and (D.35)).

Consider first the north poles. There are n northpoles and the airgap at the k 'th pole becomes:

$$\underline{k=1,2,\dots,n} \quad h_{Nk} = C [1 - \epsilon \cos(\omega t - \alpha + \frac{2\pi}{n}(k-1) + \gamma)] \quad (E.4)$$

where ω is the angular speed, C the radial clearance, α is the attitude angle and ϵ is the eccentricity ratio. The equation is derived in Appendix I, eq. (A.11), the only difference being the inclusion of the angle γ to take into account the variation in airgap along the pole face. The rotor displacement

in the plane of the northpoles are:

$$x_N = C \cos \alpha \quad (E.5)$$

$$y_N = C \sin \alpha$$

The flux density B_{Nk} is given by:

$$B_{Nk} = \frac{\mu \mathfrak{I}_N}{h_{Nk}} = B_0 \frac{[1 - f_d \cos(n\gamma) - f_q \sin(n\gamma)]}{[1 - \epsilon \cos(\omega t + \alpha + \frac{2\pi}{n}(k-1))]} \quad (E.6)$$

where:

$$B_0 = \frac{\mu \mathfrak{I}_d}{2C} \quad (E.7)$$

B_0 represents the average flux density if there is no generator load (see eq. (A.31), Appendix I).

When the pole length is l , the radius of the pole face is r , and the pole extends over an angle β , the radial force pulling on the rotor at the k 'th pole becomes:

$$\text{radial force} = \bar{Q} r l \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} B_{Nk}^2 dy \quad (E.8)$$

where \bar{Q} is a numerical conversion factor. The corresponding x and y components of the force are:

$$\begin{aligned} (F_{Nx})_k \\ (F_{Ny})_k \end{aligned} = \bar{Q} r l \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} B_{Nk}^2 \left\{ \begin{array}{l} \cos(\omega t + \frac{2\pi}{n}(k-1) + \gamma) \\ \sin(\omega t + \frac{2\pi}{n}(k-1) + \gamma) \end{array} \right\} dy \quad (E.9)$$

Since the eccentricity ratio $\epsilon \ll 1$, B_{Nk}^2 can be found from eqs. (E.6) and (E.5) as:

$$B_{Nk}^2 = \frac{B_0^2}{C} [1 - f_d \cos(n\gamma) - f_q \sin(n\gamma)]^2 [1 + 2x_N \cos(\omega t + \frac{2\pi}{n}(k-1) + \gamma) + 2y_N \sin(\omega t + \frac{2\pi}{n}(k-1) + \gamma)] \quad (E.10)$$

where:

$$[1-f_1 \cos(\eta) - f_2 \sin(\eta)]^2 = 1 + f_1^2 + f_2^2 - 2f_1 \cos(\eta) - 2f_2 \sin(\eta) + \frac{1}{2}(f_1^2 - f_2^2) \cos(2\eta) + f_1 f_2 \sin(2\eta) \quad (\text{E.11})$$

For simplification, set:

$$\eta = \omega t + \frac{2\pi}{n}(k-1)$$

Then, by substitution of eq. (E.10) into eq. (E.9):

$$\begin{aligned} \left(F_{Nx} \right)_k &= Q \frac{B_2^2 r l}{C} \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} [1-f_1 \cos(\eta) - f_2 \sin(\eta)]^2 \left\{ \begin{array}{l} [\cos(\eta + \gamma) + x_N(1 + \cos 2(\eta + \gamma)) + y_N \sin 2(\eta + \gamma)] \\ [\sin(\eta + \gamma) + x_N \sin 2(\eta + \gamma) + y_N(1 - \cos 2(\eta + \gamma))] \end{array} \right\} d\gamma \quad (\text{E.12}) \end{aligned}$$

The total forces acting on the rotor in the plane of the north poles are:

$$F_{Nx} = \sum_{k=1}^n (F_{Nx})_k \quad (\text{E.13})$$

$$F_{Ny} = \sum_{k=1}^n (F_{Ny})_k$$

As shown in Appendix I:

$$\sum_{k=1}^n \cos(k \frac{2\pi}{n}) = \sum_{k=1}^n \sin(k \frac{2\pi}{n}) = \sum_{k=1}^n \sin(k \frac{4\pi}{n}) = 0 \quad (\text{E.14})$$

$$\sum_{k=1}^n \cos(k \frac{4\pi}{n}) = \begin{cases} 2 & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases} \quad (\text{E.15})$$

Hence:

$$\sum_{k=1}^n \cos \eta = \sum_{k=1}^n \sin \eta = 0 \quad (\text{E.16})$$

$$\sum_{k=1}^n \cos 2\eta = \begin{cases} 2 \cos(2\omega t) & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases} \quad (\text{E.17})$$

$$\sum_{k=1}^n \sin 2\eta = \begin{cases} 2 \sin(2\omega t) & \text{for } n=2 \\ 0 & \text{for } n \geq 3 \end{cases} \quad (\text{E.18})$$

Therefore, for n=2 eq. (E.12) may be summed to give:

$$\left. \begin{aligned} F_{Nx} \\ F_{Ny} \end{aligned} \right\} = \bar{Q} \frac{B_0^2 r l}{C} \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1 - f_d \cos(ny) - f_q \sin(ny) \right]^2 \left\{ \begin{aligned} nx_N + 2(x_N \cos 2\omega t + y_N \sin 2\omega t) \cos y - 2(x_N \sin 2\omega t - y_N \cos 2\omega t) \sin y \\ ny_N + 2(x_N \sin 2\omega t - y_N \cos 2\omega t) \cos^2 y + 2(x_N \cos 2\omega t + y_N \sin 2\omega t) \sin^2 y \end{aligned} \right\} dy \quad (E.19)$$

Substituting from eq. (E.11), the following integrals can be evaluated:

$$I_0 = \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1 - f_d \cos(ny) - f_q \sin(ny) \right]^2 dy = \left(1 + \frac{1}{2} f_d^2 + \frac{1}{2} f_q^2 \right) \beta + \frac{1}{2n} (f_d^2 - f_q^2) \sin(n\beta) - \frac{1}{n} f_q \sin\left(\frac{n\beta}{2}\right) \quad (E.20)$$

$$I_1 = \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1 - f_d \cos(2y) - f_q \sin(2y) \right]^2 \cos 2y dy = \left(1 + \frac{1}{2} f_d^2 + \frac{1}{2} f_q^2 \right) \sin \beta + \frac{1}{4} (f_d^2 - f_q^2) \left(\frac{1}{3} \sin 3\beta + \sin \beta \right) - f_d (\beta + \frac{1}{2} \sin 2\beta) \quad (E.21)$$

$$I_2 = - \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} \left[1 - f_d \cos(2y) - f_q \sin(2y) \right]^2 \sin 2y dy = f_q \left(\beta - \frac{1}{2} \sin 2\beta \right) + \frac{1}{2} f_d f_q \left(\frac{1}{3} \sin 3\beta - \sin \beta \right) \quad (E.22)$$

In total, eq. (E.19) then becomes:

for n=2

$$F_{Nx} = 2 \bar{Q} \frac{B_0^2 r l}{C} \left[(I_0 + I_1 \cos(2\omega t) + I_2 \sin(2\omega t)) x_N + (-I_2 \cos(2\omega t) + I_1 \sin(2\omega t)) y_N \right] \quad (E.23)$$

$$F_{Ny} = 2 \bar{Q} \frac{B_0^2 r l}{C} \left[(-I_2 \cos(2\omega t) + I_1 \sin(2\omega t)) x_N + (I_0 - I_1 \cos(2\omega t) - I_2 \sin(2\omega t)) y_N \right] \quad (E.24)$$

for n=3

$$F_{Nx} = n \bar{Q} \frac{B_0^2 r l}{C} I_0 x_N \quad (E.25)$$

$$F_{Ny} = n \bar{Q} \frac{B_0^2 r l}{C} I_0 y_N \quad (E.26)$$

In the case where the angular extension β of the pole is sufficiently small that $\sin \beta \approx \beta$ and $\sin(n\beta) \approx n\beta$ eqs. (E.20) to (E.22) reduce to:
for $\sin(n\beta) \approx n\beta$

$$I_0 = I_1 = \beta (1 - f_d^2)^2 \quad (E.27)$$

$$I_2 = 0$$

whereby eqs. (E.23) to (E.26) become:

for $n=2$

$$F_{Nx} = 2 \bar{Q} \frac{B_a^2 A}{C} (1 - f_d^2)^2 [x_N (1 + \cos(2\omega t)) + y_N \sin(2\omega t)] \quad (E.28)$$

$$F_{Ny} = 2 \bar{Q} \frac{B_a^2 A}{C} (1 - f_d^2)^2 [x_N \sin(2\omega t) + y_N (1 - \cos(2\omega t))] \quad (E.29)$$

for $n \geq 3$

$$F_{Nx} = n \bar{Q} \frac{B_a^2 A}{C} (1 - f_d^2) x_N \quad (E.30)$$

$$F_{Ny} = n \bar{Q} \frac{B_a^2 A}{C} (1 - f_d^2) y_N \quad (E.31)$$

where $A = lr\beta$ is the pole area. These equations are the same as for the no-load case investigated in Appendix I except for the factor $(1 - f_d^2)^2$. In Appendix I is also given the information on preparing the corresponding input for the rotor stability and the rotor response programs.

Comparing eqs. (E.28) to (E.31) with eqs. (A.32) to (A.35) in Appendix I, it is seen that the armature reaction reduces the magnetic forces but has no other effect.

APPENDIX VI: Magnetic Forces of a Heteropolar Inductor Generator - Modifications
Due to Generator Load

In Appendix II, the magnetic forces of a heteropolar generator have been derived for the case of no load on the generator. In the case where the generator is loaded, there will be an armature reaction in form of a reverse flux set up by the current in the armature coils. This reverse flux modifies the flux due to the field coils and, hence, modifies the magnetic forces.

The generator is shown schematically in Fig. 2. Consider first the k'th pole of the generator:

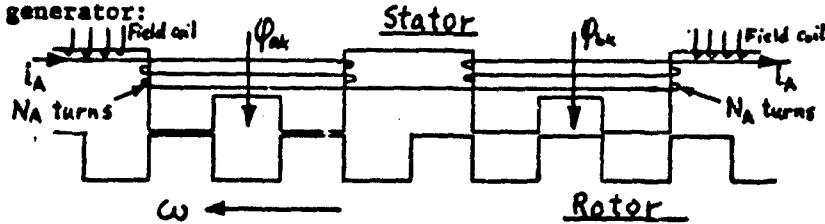
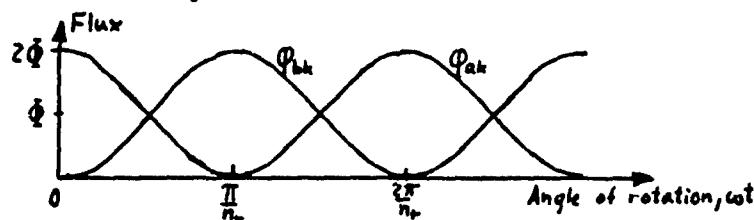


Figure 17: Windings of the k'th Pole of the Generator

The coil has two windings per pole as shown. The flux generated by the field coils passes the pole through the two windings such that the flux through one winding is Φ_{ak} and the other winding Φ_{bk} . If there are n_r rotor teeth in total, then the two flux components become:

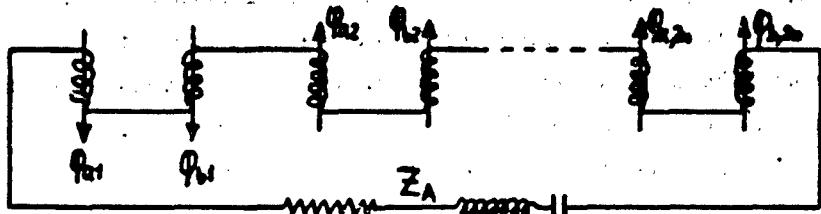


$$\Phi_{ak} = \Phi [1 + \cos(\nu t)] \quad (F.1)$$

$$\Phi_{bk} = \Phi [1 - \cos(\nu t)] \quad (F.2)$$

$$\nu = n_r \omega \quad (F.3)$$

Thus the electric frequency of the single phase output voltage is $V = n_s \omega$.
 Let the generator be connected to an output circuit with impedance Z_A .
 With n north poles and n south poles the circuit diagram is then:



The current produced in the circuit is i_A which is determined from:

$$Z_A i_A + N_A \sum_{k=1}^{2n} \left[\frac{d\Phi_{Nk}}{dt} - \frac{d\Phi_{Sk}}{dt} \right] = 0 \quad (\text{F.4})$$

where N_A is the number of turns of one winding of an armature coil.

The reluctances of the airgaps at the k 'th pole are R_{Nk} and R_{Sk} respectively.
 They are given by (see Appendix II):

$$\frac{1}{R_{Nk}} = (1 + \cos(\nu t)) [P + \epsilon(G_k - H_k)] \quad (\text{F.5})$$

$$\frac{1}{R_{Sk}} = (1 - \cos(\nu t)) [P + \epsilon(G_k + H_k)] \quad (\text{F.6})$$

where:

$$P = \frac{\mu A_T n_s}{2C} \quad (\text{F.7})$$

$$G_k = \frac{\mu A_T}{2C} \cos\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) \cdot \sum_{j=1}^{n_s} \cos((4j-1)\frac{\pi}{2n_r}) \quad (\text{F.8})$$

$$H_k = \frac{\mu A_T}{2C} \sin\left(\frac{\pi}{n}(k-1) + \frac{\pi}{2n} - \alpha\right) \cdot \sum_{j=1}^{n_s} \sin((4j-1)\frac{\pi}{2n_r}) \quad (\text{F.9})$$

where A_T is the area of a stator tooth, μ is the permeability of air, C is the radial airgap, n is the number of north poles (= number of south poles),

ϵ is the eccentricity ratio of the rotor with respect to the stator and α is the corresponding attitude angle. Furthermore, there are $2n_1$ stator teeth per pole.

When the mmf across the airgap is f , the flux for the two sections of the k^{th} pole becomes:

$$\Phi_{ak} = \frac{1}{R_{ak}} f = (1 + \cos(\nu t)) [P + \epsilon (G_k - H_k)] f \quad (\text{F.10})$$

$$\Phi_{bk} = \frac{1}{R_{bk}} f = (1 - \cos(\nu t)) [P + \epsilon (G_k + H_k)] f \quad (\text{F.11})$$

whereby:

$$\sum_{k=1}^{2n} \left(\frac{d\Phi_{ak}}{dt} - \frac{d\Phi_{bk}}{dt} \right) = 2 \frac{d}{dt} \left\{ \sum_{k=1}^{2n} [(P + \epsilon G_k) \cos(\nu t) - \epsilon H_k] f \right\} \quad (\text{F.12})$$

The following identities hold true:

$$\sum_{k=1}^{2n} \cos\left(\frac{\pi}{n} k\right) = 0 \quad (\text{F.13})$$

$$\sum_{k=1}^{2n} \sin\left(\frac{\pi}{n} k\right) = 0 \quad (\text{F.13})$$

from which follows:

$$\sum_{k=1}^{2n} G_k = \sum_{k=1}^{2n} H_k = 0 \quad (\text{F.14})$$

i.e.:

$$\sum_{k=1}^{2n} \left(\frac{d\Phi_{ak}}{dt} - \frac{d\Phi_{bk}}{dt} \right) = 4n P \frac{d}{dt} (f \cos(\nu t)) \quad (\text{F.15})$$

The mmf across a winding of an armature coil is $N_A i_A$ which can be found from eq. (F.4) by substitution from eq. (F.15):

$$N_A i_A = - \frac{N_A^2}{Z_A} 4n P \frac{d}{dt} (f \cos(\nu t)) \quad (\text{F.16})$$

Since this mmf is independent of rotor eccentricity, the mmf across all poles will be the same and equal to $\frac{1}{2} \mathcal{F}_f$ per pole where \mathcal{F}_f is the mmf of a field coil (see Appendix II). Thus, the mmf across the k'th pole can be expressed in terms of its components:

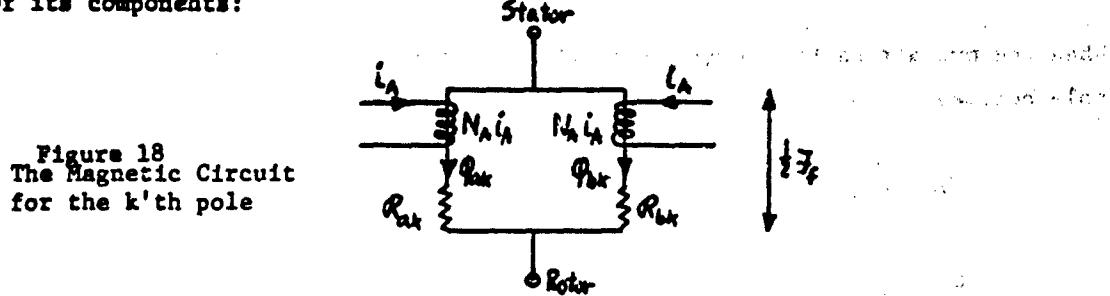


Figure 18
The Magnetic Circuit
for the k'th pole

$$\frac{1}{2} \mathcal{F}_f = N_A i_A + R_{ek} \phi_{ek} = N_A i_A + R_{bk} \phi_{bk} = N_A i_A + f = f - 4n_P \frac{N_A^2}{Z_A dt} (f_{asym}) \quad (F.17)$$

Consider next the k'th field coil, located between the (k-1)'th and the k'th pole. The coil has N_f turns with a flux ϕ_k passing through it. The field coil circuit has an impressed d.c. voltage E_f , an impedance Z_f and a current i_f . Hence, the equation for the field coil is:

$$Z_f i_f + N_f \frac{d\phi_k}{dt} = E_f \quad (F.18)$$

from which the mmf of the field coil becomes:

$$\mathcal{F}_f = N_f i_f = \frac{N_f}{R_f} E_f - \frac{N_f^2}{Z_f} \frac{d\phi_k}{dt} \quad (F.19)$$

where R_f is the resistance of the field coil circuit.

The total flux through the k'th pole is ($\phi_{ek} + \phi_{bk}$). Since the flux ϕ_k from each field coil passes through two poles, a summation over all field coils and all poles yields:

$$\phi = \sum_{k=1}^{2n} \phi_k = \frac{1}{2} \sum_{k=1}^{2n} (\phi_{ek} + \phi_{bk}) \quad (F.20)$$

Hence, summing eq. (F.19) from k=1 to k=2n, the result becomes:

$$2n \mathcal{F}_f = 2n \frac{N_f}{R_f} E_f - \frac{N_f^2}{Z_f} \sum_{k=1}^{2n} \frac{d\phi_k}{dt} \quad (F.21)$$

or:

$$\bar{Z}_f = \frac{N_f}{R_f} E_f - \frac{1}{4n} \frac{N_f^2}{\bar{Z}_f} \frac{d}{dt} \left[\sum_{k=1}^{2n} (\varphi_{ak} + \varphi_{bk}) \right] \quad (F.22)$$

Substitute for φ_{ak} and φ_{bk} from eqs. (F.10) and (F.11):

$$\varphi_{ak} + \varphi_{bk} = [2(P + \epsilon C_k) - 2\epsilon H_k \cos(\gamma t)] f$$

or making use of eq. (F.14):

$$\sum_{k=1}^{2n} (\varphi_{ak} + \varphi_{bk}) = 4n Pf \quad (F.23)$$

whereby eq. (F.22) becomes:

$$\bar{Z}_f = \frac{N_f}{R_f} E_f - P \frac{N_f^2}{\bar{Z}_f} \frac{df}{dt} \quad (F.24)$$

Equate eqs. (F.17) and (F.24) to get:

$$\bar{Z}_f = \frac{N_f}{R_f} E_f - P \frac{N_f^2}{\bar{Z}_f} \frac{df}{dt} = 2f - 8nP \frac{N_f^2}{\bar{Z}_A} \frac{d}{dt} (f \cos(\gamma t)) \quad (F.25)$$

which is a first order, ordinary differential equation in the variable f . If \bar{Z}_f and \bar{Z}_A are pure resistances, this equation has a closed form solution which, however, is not too convenient for the present purposes anyway. Instead, f shall be expressed as a Fourier series:

$$f = f_0 \left[1 + \sum_{m=1}^{\infty} (f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi)) \right] \quad (F.26)$$

where:

$$\psi = \gamma t \quad (F.27)$$

Hence:

$$f \cos(\gamma t) = f \cos(\psi) = f_0 \left[\cos \psi + \frac{1}{2} \sum_{m=1}^{\infty} [f_{cm} (\cos(m+1)\psi + \cos(m-1)\psi) - f_{sm} (\sin(m+1)\psi + \sin(m-1)\psi)] \right] \quad (F.28)$$

Set:

$$f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi) = f_m e^{im\psi} \quad (\text{F.29})$$

where:

$$\begin{aligned} f_m &= f_{cm} + i f_{sm} \\ i &= \sqrt{-1} \end{aligned} \quad (\text{F.30})$$

and it is understood that only the real part applies. Hence:

$$f = f_0 \left[1 + \sum_{m=1}^{\infty} f_m e^{im\psi} \right] \quad (\text{F.31})$$

$$f \cos(\nu t) = f_0 \left[e^{i\psi} + \frac{1}{2} \sum_{m=1}^{\infty} f_m (e^{i(m-1)\psi} + e^{i(m+1)\psi}) \right] \quad (\text{F.32})$$

Next, consider the impedances Z_A and Z_f :

$$\frac{1}{Z_A} \cong \frac{1}{R_A + i(m\nu L_A - \frac{1}{m\nu C_A})} = Y_{Am} \quad (\text{F.33})$$

$$\frac{1}{Z_f} \cong \frac{1}{R_f + i(m\nu L_f - \frac{1}{m\nu C_f})} = Y_{fm} \quad (\text{F.34})$$

where R is electrical resistance, C is capacitance and L is inductance (L does not include the field coils or the armature coils). Actually, the circuits may have more than one resonance but they can still be represented by frequency dependent impedances. Thus:

$$\frac{1}{Z_f} \frac{df}{dt} = i\nu f_0 \sum_{m=1}^{\infty} m Y_{fm} f_m e^{im\psi} \quad (\text{F.35})$$

$$\frac{1}{Z_A} \frac{d}{dt} (f \cos \psi) = i\nu f_0 \left\{ Y_{A1} e^{i\psi} + \frac{1}{2} \sum_{m=1}^{\infty} f_m [(m-1) Y_{A,m-1} e^{i(m-1)\psi} + (m+1) Y_{A,m+1} e^{i(m+1)\psi}] \right\} \quad (\text{F.36})$$

Substituting eqs. (F.31), (F.35) and (F.36) into eq. (F.25) and collecting terms according to powers of $e^{im\psi}$, an infinite set of simultaneous equations are obtained:

$$\underline{m=0} \quad f_0 = \frac{1}{2} N_s \frac{1}{R_f} E_f \quad (\text{F.37})$$

$$\underline{m=1} \quad [2 + i\mu Y_{f1} P N_f^2] f_1 - i4\mu Y_{A1} n P N_A^2 f_2 = i8\mu Y_{A1} n P N_A^2 \quad (\text{F.38})$$

$$\underline{m=2} \quad -i4\mu Y_{A2} n P N_A^2 f_{m+1} + [2 + i\mu Y_{fm} P N_f^2] f_m - i4\mu Y_{Am} n P N_A^2 f_{m+1} = 0 \quad (\text{F.39})$$

Define:

$$x_m = 2 + i\mu Y_{fm} P N_f^2 \quad (\text{F.40})$$

$$\lambda_m = i4\mu Y_{Am} n P N_A^2 \quad (\text{F.41})$$

As $m \rightarrow \infty$, $i\mu Y_{fm}$ becomes $\frac{1}{L_f}$ and $i\mu Y_{Am}$ becomes $\frac{1}{L_A}$ (see eqs. (F.33) and (F.34)), i.e.:

$$\begin{aligned} x_m &\rightarrow 2 + \frac{P N_f^2}{L_f} \\ \lambda_m &\rightarrow \frac{4n P N_A^2}{L_A} \end{aligned} \quad (\text{F.42})$$

Substituting eqs. (F.40) and (F.41) into eq. (F.39):

$$\underline{m \geq 2} \quad -\lambda_m f_{m-1} + x_m f_m - \lambda_m f_{m+1} = 0 \quad (\text{F.43})$$

Assume, that eq. (F.43) reduces to identities for all $m \geq M$, and add all the corresponding eqs. (F.43) for $m \geq M$ to get:

$$(x_M - 2\lambda_M) \sum_{m=M}^{\infty} f_m = \lambda_{M+1} f_{M+2} - (x_{M+1} - \lambda_M) f_{M+1} \quad (\text{F.44})$$

from which it is concluded, that since the sum $\sum_{m=M}^{\infty} f_m$ which contains infinite many terms, has a finite value, each term f_m must be small. This is, of course, an inadequate proof from a mathematical point of view but is sufficient from a physical point of view. Thus, all higher harmonics of f may be ignored but before solving for f , the magnetic forces acting on the rotor will

be derived.

The magnetic forces are F_x and F_y . They have been determined in Appendix II for the case of no load where the mmf across the airgaps at the poles is equal to $\frac{1}{2} \bar{\mathcal{I}}_0$ ($\bar{\mathcal{I}}_0 = \frac{N_f}{R_f} E_f$). In the present case of a loaded generator this mmf is equal to f . Thus, substituting f for $\frac{1}{2} \bar{\mathcal{I}}_0$ in eqs. (B.36) and (B.37) of Appendix II, the magnetic forces become:

$$F_x = \bar{Q} \frac{2nn_s A_T \mu^2}{C^3} f^2 x \quad (F.45)$$

$$F_y = \bar{Q} \frac{2nn_s A_T \mu^2}{C^3} f^2 y \quad (F.46)$$

The flux density B_0 in the case of the unloaded generator is:

$$B_0 = \frac{\mu \frac{1}{2} \bar{\mathcal{I}}_0}{C} = \frac{\mu N_f E_f}{2 C R_f} = \frac{\mu f_0}{C} \quad (F.47)$$

whereby eqs. (F.45) and (F.46) can be written:

$$F_x = \bar{Q} \frac{2nn_s A_T B_0^2}{C} \left(\frac{f}{f_0} \right)^2 x \quad (F.48)$$

$$F_y = \bar{Q} \frac{2nn_s A_T B_0^2}{C} \left(\frac{f}{f_0} \right)^2 y \quad (F.49)$$

Here, $\frac{f}{f_0}$ is given by eq. (F.26):

$$\frac{f}{f_0} = 1 + \sum_{m=1}^{\infty} (f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi)) \quad (F.50)$$

from which:

$$\left(\frac{f}{f_0} \right)^2 = 1 + 2 \sum_{m=1}^{\infty} (f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi)) + \sum_{m=1}^{\infty} \left\{ (f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi)) \sum_{l=1}^{\infty} (f_{cl} \cos(l\psi) - f_{sl} \sin(l\psi)) \right\} \quad (F.51)$$

Making use of the trigonometric identities:

$$\begin{aligned}\cos(m\psi) \cos(l\psi) &= \frac{1}{2} [\cos(m+l)\psi + \cos(m-l)\psi] \\ \cos(m\psi) \sin(l\psi) &= \frac{1}{2} [\sin(m+l)\psi - \sin(m-l)\psi] \\ \sin(m\psi) \cos(l\psi) &= \frac{1}{2} [\sin(m+l)\psi + \sin(m-l)\psi] \\ \sin(m\psi) \sin(l\psi) &= \frac{1}{2} [\cos(m-l)\psi - \cos(m+l)\psi]\end{aligned}\quad (F.52)$$

eq. (F.51) can also be written:

$$\left(\frac{f}{f_0}\right)^2 = 1 + 2 \sum_{m=1}^{\infty} (f_{cm} \cos(m\psi) - f_{sm} \sin(m\psi)) + \frac{1}{2} \sum_{m=1}^{\infty} \sum_{l=1}^{\infty} \left\{ f_{cm} f_{cl} [\cos(m+l)\psi + \cos(m-l)\psi] \right. \\ \left. - f_{sm} f_{sl} [\cos(m+l)\psi - \cos(m-l)\psi] - f_{sm} f_{cl} [\sin(m+l)\psi + \sin(m-l)\psi] - f_{cm} f_{sl} [\sin(m+l)\psi - \sin(m-l)\psi] \right\} \quad (F.53)$$

Collect terms $\sin(m\psi)$ and $\cos(m\psi)$ to get:

$$\begin{aligned}\left(\frac{f}{f_0}\right)^2 = 1 + \frac{1}{2} \sum_{m=1}^{\infty} (f_{cm}^2 + f_{sm}^2) + [2f_{c1} + \sum_{m=1}^{\infty} (f_{cm} f_{c,m+1} + f_{sm} f_{s,m+1})] \cos\psi \\ - [2f_{s1} + \sum_{m=1}^{\infty} (f_{cm} f_{s,m+1} - f_{sm} f_{c,m+1})] \sin\psi \\ + \sum_{m=2}^{\infty} \left\{ [2f_{cm} + \sum_{l=1}^{\infty} (f_{cl} f_{c,m+l} + f_{sl} f_{s,m+l})] + \frac{1}{2} \sum_{l=1}^{m-1} (f_{cl} f_{c,m-l} - f_{sl} f_{s,m-l}) \right\} \cos(m\psi) \\ - \left\{ 2f_{sm} + \sum_{l=1}^{\infty} (f_{cl} f_{s,m+l} - f_{sl} f_{c,m+l}) + \frac{1}{2} \sum_{l=1}^{m-1} (f_{cl} f_{s,m-l} + f_{sl} f_{c,m-l}) \right\} \sin(m\psi) \quad (F.54)\end{aligned}$$

Substitution of this equation into eqs. (F.48) and (F.49) results in the final expressions for the magnetic forces.

Returning to the solution of eqs. (F.38) and (F.39), they define an infinite set of simultaneous equations

$$\left\{ \begin{array}{ccccccc} x_1 & -\lambda_1 & 0 & 0 & 0 & \cdots & \\ -\lambda_2 & x_2 & -\lambda_2 & 0 & 0 & \cdots & \\ 0 & -\lambda_3 & x_3 & -\lambda_3 & 0 & \cdots & \\ 0 & 0 & -\lambda_4 & x_4 & -\lambda_4 & \cdots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \end{array} \right\} \left\{ \begin{array}{c} f_1 \\ f_2 \\ f_3 \\ f_4 \\ \vdots \\ \vdots \end{array} \right\} = \left\{ \begin{array}{c} 2\lambda_1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \end{array} \right\} \quad (F.55)$$

where x_m and λ_m are complex numbers defined by eqs. (F.40) and (F.41). For $m \geq 2$ these equations are of the form given by eq. (F.43). Define σ_m by the equation:

$$f_m = \sigma_{m-1} f_{m-1}, \quad (F.56)$$

and substitute into eq. (F.43) to get:

$$-\lambda_m f_{m-1} + (x_m - \lambda_m \sigma_m) f_m = 0$$

Comparing this equation with eq. (F.56) yields:

$$\underline{m=1,2,3,\dots} \quad \sigma_{m-1} = \frac{\lambda_m}{x_m - \sigma_m \lambda_m} \quad (F.57)$$

Assume that all f_m for $m > M$ are so small that they can be ignored, i.e. $f_{M+1} = f_{M+2} = \dots \approx 0$. Then, from eq. (F.56):

$$\sigma_M = 0$$

With this as a starting condition, repeated use of eq. (F.57), starting with $m=M$ makes it possible to calculate $\sigma_{M-1}, \sigma_{M-2}, \dots, \sigma_1, \sigma_0$. From the first equation of eq. (55):

$$x_1 f_1 - \lambda_1 f_2 = 2\lambda_1$$

one obtains:

$$f_1 = \frac{2\lambda_1}{x_1 - \sigma_1 \lambda_1} = 2\sigma_0 \quad (F.58)$$

after which eq. (F.56) can be used to obtain the results for the other f -values:

$$\underline{m=1,2,\dots} \quad f_m = 2\sigma_0 \cdot \sigma_1 \cdot \sigma_2 \cdot \dots \cdot \sigma_{m-1} \quad (F.59)$$

In the case where only the two first harmonics are important, the solution of eq. (55) can be found directly as:

$$f_1 = f_{c1} + i f_{s1} = \frac{2\lambda_1 x_2}{x_1 x_2 - \lambda_1 \lambda_2} \quad (\text{F.60})$$

$$f_2 = f_{c2} + i f_{s2} = \frac{2\lambda_1 \lambda_2}{x_1 x_2 - \lambda_1 \lambda_2} \quad (\text{F.61})$$

The same result can, of course, also be obtained by using the outlined general method. Here, $f_3 = f_4 = \dots = 0$, so that $M=2$ and $\delta_2 = 0$.

Then, from eq. (F.57):

$$\begin{aligned}\delta_1 &= \frac{\lambda_1}{x_2} \\ \delta_0 &= \frac{\lambda_1}{x_1 - \delta_1 \lambda_1} = \frac{\lambda_1 x_2}{x_1 x_2 - \lambda_1 \lambda_2}\end{aligned}$$

i.e.:

$$f_1 = 2\delta_0 = \frac{2\lambda_1 x_2}{x_1 x_2 - \lambda_1 \lambda_2}$$

$$f_2 = 2\delta_0 \delta_1 = \frac{2\lambda_1 \lambda_2}{x_1 x_2 - \lambda_1 \lambda_2}$$

which agrees with eqs. (F.60) and (F.61).

Hence, by truncating the equations at $m=2$, the result becomes:

$$\frac{f}{f_0} = 1 + (f_{c1} \cos(\nu t) - f_{s1} \sin(\nu t)) + (f_{c2} \cos(2\nu t) - f_{s2} \sin(2\nu t))$$

From eq. (F.54) with $f_{c3} = f_{s3} = f_{c4} = f_{s4} = \dots = 0$:

$$\begin{aligned}\left(\frac{f}{f_0}\right)^2 &= 1 + \frac{1}{2}(f_{c1}^2 + f_{s1}^2 + f_{c2}^2 + f_{s2}^2) + [2f_{c1} + f_{c1}f_{c2} + f_{s1}f_{s2}] \cos(\nu t) \\ &\quad - [2f_{s1} + f_{c1}f_{s2} - f_{s1}f_{c2}] \sin(\nu t) + [2f_{c2} + \frac{1}{2}f_{c1}^2 - \frac{1}{2}f_{s1}^2] \cos(2\nu t) \\ &\quad - [2f_{s2} + f_{c1}f_{s1}] \sin(2\nu t) + [f_{c1}f_{c2} - f_{s1}f_{s2}] \cos(3\nu t) - [f_{c1}f_{s2} + f_{s1}f_{c2}] \sin(3\nu t) \\ &\quad + \frac{1}{2}[f_{c2}^2 - f_{s2}^2] \cos(4\nu t) - f_{c2}f_{s2} \sin(4\nu t)\end{aligned} \quad (\text{F.62})$$

which then can be substituted into eqs. (F.48) and (F.49) to obtain the magnetic forces.

The response and stability computer programs consider only one frequency component of the magnetic forces. Let this be the first harmonic so that the magnetic force frequency Ω is:

$$\Omega = \nu = h_r \omega \quad (F.63)$$

i.e.

$$\frac{\Omega}{\omega} = h_r \quad (F.64)$$

Writing the magnetic forces in the form given by eq. (A.46), Appendix I, it is found that:

$$Q_0 = \bar{Q} \frac{2n n_2 A_T B_o^2}{C} \left[1 + \frac{1}{2} (f_{c1}^2 + f_{s1}^2 + f_{c2}^2 + f_{s2}^2) \right] \quad (F.65)$$

$$Q'_0 = 0 \quad (F.66)$$

$$Q = -\bar{Q} \frac{2n n_2 A_T B_o^2}{C} (2f_{c1} + f_{c1}f_{c2} + f_{s1}f_{s2}) \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Bmatrix} \quad (F.67)$$

$$q = -\bar{Q} \frac{2n n_2 A_T B_o^2}{C} (2f_{s1} + f_{c1}f_{s2} - f_{s1}f_{c2}) \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Bmatrix} \quad (F.68)$$

which shows how to prepare the input for the two programs.

APPENDIX VII: Units and Dimensions

The electric and magnetic units are summarized in this section.

Magnetomotive Force

$$\mathcal{F} = Ni \text{ amp-turns}$$

with i = electric current, amp
 N = number of turns

(G.1)

Reluctance

$$R = \frac{h}{\mu A}$$

h = length in the direction of flux, meter

A = cross-sectional area, sq. meter

μ = permeability = $4\pi \cdot 10^{-7}$ for air

(G.2)

Flux

$$\Phi = \frac{\mathcal{F}}{R} \text{ Weber} = \frac{\mathcal{F}}{R} \cdot 10^8 \text{ lines}$$

with \mathcal{F} in amp - turns
and R from eq. (G.2)

(G.3)

Induced Voltage

$$e = -N \frac{d\Phi}{dt} \text{ volt}$$

with Φ in Weber

N is the number of turns

t = time, seconds

(G.4)

Magnetic Force

$$F = \bar{Q} B^2 A \text{ lbs}$$

with $\bar{Q} = 1/72,130,000$

B in lines/in²

A in in²

(G.5)

APPENDIX VIII: The Impedance of an Arbitrary, Elastic Rotor Supported in Flexible, Damped Bearings

In determining the stability threshold and the amplitude of a rotor with time-varying magnetic forces it is necessary to calculate the response of the rotor to high frequency excitation. In the more common case of a rotor response to mechanical unbalance it is customary to neglect the contribution from shear force, but at high frequencies this contribution becomes important and must be included. Thus, the previous analysis given in Volume 5 will be extended to include the effect of shear force and, in addition, the new analysis will take into account the actual mass distribution along the rotor.

Referring to Figure 4, let the rotor be subdivided into sections such that each shaft section has uniform diameter and uniform material properties. The endpoints of the sections are called rotor stations. Stations are introduced not only where the shaft diameter changes, but also where there are concentrated masses like wheels, impellers or sleeves, where there are bearings, where there are magnetic forces and at the endpoints of the rotor. Hence, the arbitrary rotor station n can be assigned a mass m_n , a polar and a transverse mass moment of inertia, I_{Pn} and I_{Tn} , 8 bearing coefficients: $K_{xx,n}, B_{xxn}, K_{xy,n}, B_{xy,n}, K_{yx,n}, B_{yxn}, K_{yy,n}$ and B_{ynn} , and magnetic forces (some or all of these quantities may be zero at any particular station). This results in an abrupt change in both the bending moment M and in the shear force V across a rotor station.

Introduce a cartesian coordinate system with the Z -axis along the rotor, the x -axis vertical downwards and the y -axis horizontal. At each point along the rotor the origin of the coordinate system coincides with the steady-state position of the rotor axis. The rotor amplitudes, caused by the applied dynamic forces, are therefore x and y . They are functions of z . Consider rotor station n :

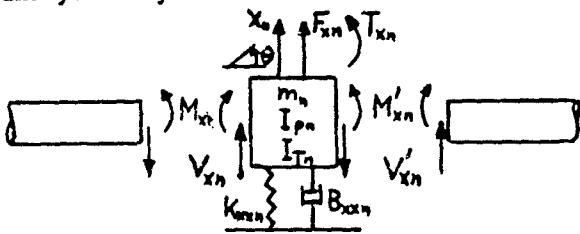


Figure 19: Force Diagram for Rotor Station n

where Θ_n is the slope of the bent rotor (in the y-plane the slope is Φ_n),

F_{xn} is the x-component of the externally applied force and T_{xn} is the x-component of the externally applied moment. A force balance yields:

$$m_n \frac{d^2x_n}{dt^2} = V'_{xn} - V_{xn} - (K_{xxn} - Q_0)x_n - B_{xxn} \frac{dx_n}{dt} - K_{xy_n} y_n - B_{xy_n} \frac{dy_n}{dt} + F_{xn} \quad (\text{H.1})$$

$$m_n \frac{d^2y_n}{dt^2} = V'_{yn} - V_{yn} - K_{yy_n} x_n - B_{yy_n} \frac{dx_n}{dt} - (K_{yy_n} - Q_0)y_n - B_{yy_n} \frac{dy_n}{dt} + F_{yn} \quad (\text{H.2})$$

A moment balance yields:

$$I_{Tn} \frac{d^2\theta_n}{dt^2} + \omega I_{pn} \frac{d\phi_n}{dt} = M'_{xn} - M_{xn} + Q'_0 \Theta_n + T_{xn} \quad (\text{H.3})$$

$$I_{Tn} \frac{d^2\theta_n}{dt^2} - \omega I_{pn} \frac{d\phi_n}{dt} = M'_{yn} - M_{yn} + Q'_0 \Phi_n + T_{yn} \quad (\text{H.4})$$

where ω is the angular speed of the rotor, and Q_0 and Q'_0 are the static gradients of the magnetic force and the magnetic moment.

Assume next that the motion takes place with a given frequency ν such that:

$$x_n = x_{cn} \cos(\nu t) - x_{sn} \sin(\nu t) = \Re \{(x_{cn} + i x_{sn}) e^{i\nu t}\} \quad (\text{H.5})$$

and similarly for $y_n, \Theta_n, \Phi_n, \varphi_n, M_{xn}, M_{yn}, V_{xn}, V_{yn}, F_{xn}, F_{yn}, T_{xn}$ and T_{yn} . Hence, eqs. (H.1) to (H.4) can be written:

$$V'_{xcn} = V_{xcn} + ((\frac{\nu}{\omega})^2 m_n \omega^2 - K_{xxn} + Q_0)x_{cn} + (\frac{\nu}{\omega}) \omega B_{xxn} x_{sn} - K_{xy_n} y_{cn} + (\frac{\nu}{\omega}) \omega B_{xy_n} y_{sn} + F_{xcn} \quad (\text{H.6})$$

$$V'_{xsn} = V_{xsn} - (\frac{\nu}{\omega}) \omega B_{xxn} x_{cn} + ((\frac{\nu}{\omega})^2 m_n \omega^2 - K_{xxn} + Q_0)x_{sn} - (\frac{\nu}{\omega}) \omega B_{xy_n} y_{cn} - K_{xy_n} y_{sn} + F_{xsn} \quad (\text{H.7})$$

$$V'_{ycn} = V_{ycn} - K_{yx_n} x_{cn} + (\frac{\nu}{\omega}) \omega B_{yx_n} x_{sn} + ((\frac{\nu}{\omega})^2 m_n \omega^2 - K_{yy_n} + Q_0)y_{cn} + (\frac{\nu}{\omega}) \omega B_{yy_n} y_{sn} + F_{ycn} \quad (\text{H.8})$$

$$V'_{ysn} = V_{ysn} - (\frac{\nu}{\omega}) \omega B_{yx_n} x_{cn} - K_{yx_n} x_{sn} - (\frac{\nu}{\omega}) \omega B_{yy_n} y_{cn} + ((\frac{\nu}{\omega})^2 m_n \omega^2 - K_{yy_n} + Q_0)y_{sn} + F_{ysn} \quad (\text{H.9})$$

$$M'_{x_{cn}} = M_{x_{cn}} - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \theta_{cn} - (\dot{\omega}) I_{pn} \omega^2 \phi_{sn} - T_{x_{cn}} \quad (H.10)$$

$$M'_{y_{sn}} = M_{y_{sn}} - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \theta_{sn} + (\dot{\omega}) I_{pn} \omega^2 \phi_{cn} - T_{y_{sn}} \quad (H.11)$$

$$M'_{y_{cn}} = M_{y_{cn}} + (\dot{\omega}) I_{pn} \omega^2 \theta_{sn} - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \phi_{cn} - T_{y_{cn}} \quad (H.12)$$

$$M'_{x_{sn}} = M_{x_{sn}} - (\dot{\omega}) I_{pn} \omega^2 \theta_{cn} - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \phi_{sn} - T_{x_{sn}} \quad (H.13)$$

This can also be written in a more compact form by expressing all the quantities in the same way as in eq. (H.5) and with the convention that only the real part applies:

$$V'_{x_n} = V_{x_n} + \left[(\dot{\omega})^2 m_n \omega^2 - K_{xx_n} + Q_o - i(\dot{\omega}) \omega B_{xx_n} \right] x_n - (K_{xy_n} + i(\dot{\omega}) \omega B_{xy_n}) y_n + F_{x_n} \quad (H.14)$$

$$V'_{y_n} = V_{y_n} - (K_{yx_n} + i(\dot{\omega}) \omega B_{yx_n}) x_n + \left[(\dot{\omega})^2 m_n \omega^2 - K_{yy_n} + Q_o - i(\dot{\omega}) \omega B_{yy_n} \right] y_n + F_{y_n} \quad (H.15)$$

$$M'_{x_n} = M_{x_n} - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \theta_n + i(\dot{\omega}) I_{pn} \omega^2 \phi_n - T_{x_n} \quad (H.16)$$

$$M'_{y_n} = M_{y_n} - i(\dot{\omega}) I_{pn} \omega^2 \theta_n - \left[(\dot{\omega})^2 I_{Tn} \omega^2 + Q'_o \right] \phi_n - T_{y_n} \quad (H.17)$$

Having established the change in shear force and bending moment across a rotor station (eqs. (H.6) to (H.13) or eqs. (H.14) to (H.17)), the shaft sections connecting the rotor stations will be considered. The governing equations are (ref. 1):

$$\text{shaft deflection: } \frac{\partial x}{\partial z} = \theta - \frac{V_x}{\alpha A G} \quad (H.19)$$

$$\text{shaft bending: } \frac{\partial \theta}{\partial z} = \frac{1}{EI} M_x \quad (F.20)$$

$$\text{force balance: } gA \frac{\partial^2 x}{\partial z^2} = - \frac{\partial V_x}{\partial z} \quad (H.21)$$

$$\text{moment balance: } \frac{\partial M_x}{\partial z} = V_x \quad (H.22)$$

where x is the amplitude, θ is the rotation angle, M is the bending moment, V is the shear force, A is the cross-sectional area, I is the transverse moment of inertia of cross-section, ρ is the mass density, E is Young's modulus, G is the shear modulus and α is a shape factor for shear ($\alpha \approx 0.75$ for circular cross-section). Analogous equations hold for the y -direction. It should be noted that rotary inertia and gyroscopic moments have been ignored in the last equation above because these contributions are rather small and can be accounted for at the rotor stations.

These equations can be combined. Substitute for V_x from eq. (H.22) into eq. (H.19) and differentiate with respect to z :

$$\frac{dx}{dz^2} = \frac{d\theta}{dz} - \frac{1}{\alpha AG} \frac{d^2M_x}{dz^2} \quad (\text{H.23})$$

Substitute for $\frac{d\theta}{dz}$ from eq. (H.20) and differentiate twice with respect to t :

$$\frac{d^4x}{dz^2 dt^2} = \frac{1}{EI} \frac{d^2M_x}{dt^2} - \frac{1}{\alpha AG} \frac{d^4M_x}{dz^2 dt^2} \quad (\text{H.24})$$

Next, differentiate eq. (H.22) with respect to z and substitute into eq. (H.21):

$$gA \frac{d^2x}{dt^2} = - \frac{d^2M_x}{dz^2} \quad (\text{H.25})$$

or:

$$\frac{d^4x}{dz^2 dt^2} = - \frac{1}{gA} \frac{d^4M_x}{dz^4} \quad (\text{H.26})$$

By equating the two expressions for $\frac{d^4x}{dz^2 dt^2}$, the final equation becomes:

$$\frac{d^4M_x}{dz^4} - \frac{gA}{\alpha AG} \frac{d^4M_x}{dz^2 dt^2} + \frac{gA}{EI} \frac{d^2M_x}{dt^2} = 0 \quad (\text{H.27})$$

Let the motion be harmonic with frequency ν , i.e. $\frac{d^2M_x}{dt^2} = -\nu^2 M_x$. Furthermore, define:

$$\beta^4 = \frac{\nu^2 A}{EI} \quad (\text{H.28})$$

$$\delta^2 = \frac{EI}{2\alpha AG} \quad (\text{H.29})$$

Thereby eq. (H.27) can be written:

$$\frac{d^4 M_x}{dz^4} + 2\delta^2 \beta^4 \frac{d^2 M_x}{dz^2} - \beta^4 M_x = 0 \quad (\text{H.30})$$

The characteristic equation is:

$$s^4 + 2\delta^2 \beta^4 s^2 - \beta^4 = 0 \quad (\text{H.31})$$

with the roots:

$$s^2 = -\delta^2 \beta^4 \pm \sqrt{\beta^4 + (\delta^2 \beta^4)^2} = \beta^2 [-(\delta \beta)^2 \pm \sqrt{1 + (\delta \beta)^4}]$$

Set:

$$\left. \begin{array}{l} \beta_1 \\ \beta_2 \end{array} \right\} = \beta [\sqrt{1 + (\delta \beta)^4} \mp (\delta \beta)^2]^{1/2} \quad (\text{H.32})$$

whereby the four roots become:

$$s_1 = \beta_1, \quad s_2 = -\beta_1, \quad s_3 = i\beta_2, \quad s_4 = -i\beta_2$$

and the final solution can be written:

$$\frac{1}{EI} M_x = C_1 \cosh(\beta_1 z) + C_2 \sinh(\beta_1 z) + C_3 \cos(\beta_2 z) + C_4 \sin(\beta_2 z) \quad (\text{H.33})$$

The three other variables become:

$$\frac{1}{EI} V_x = \frac{1}{EI} \frac{dM_x}{dz} = C_1 \beta_1 \sinh(\beta_1 z) + C_2 \beta_1 \cosh(\beta_1 z) - C_3 \beta_2 \sin(\beta_2 z) + C_4 \beta_2 \cos(\beta_2 z) \quad (\text{H.34})$$

$$x = \frac{1}{\nu^3 A} \frac{dV_x}{dz} = \frac{1}{\beta^4 EI} \frac{dV_x}{dz} = \frac{C_1}{\beta_1^2} \cosh(\beta_1 z) + \frac{C_2}{\beta_1^2} \sinh(\beta_1 z) - \frac{C_3}{\beta_2^2} \cos(\beta_2 z) - \frac{C_4}{\beta_2^2} \sin(\beta_2 z) \quad (\text{H.35})$$

$$\Theta = \int \frac{M_x}{EI} dz = \frac{C_1}{\beta_1} \sinh(\beta_1 z) + \frac{C_2}{\beta_1} \cosh(\beta_1 z) + \frac{C_3}{\beta_2} \sin(\beta_2 z) - \frac{C_4}{\beta_2} \cos(\beta_2 z) \quad (H.36)$$

For the shaft section of length l_n between rotor stations n and $(n+1)$, the end conditions are:

$$\text{at } z=0 \quad x=x_n \quad \theta=\theta_n \quad M_x=M'_{x_n} \quad V_x=V'_{x_n}$$

$$\text{at } z=l_n \quad x=x_{n+1} \quad \theta=\theta_{n+1} \quad M_x=M'_{x_{n+1}} \quad V_x=V'_{x_{n+1}}$$

Set:

$$\lambda_1 = l_n \beta_1, \quad \lambda_2 = l_n \beta_2 \quad (H.37)$$

Then the four constants C_1 , C_2 , C_3 , and C_4 are determined from the equations:

$$C_1 + C_3 = \frac{1}{EI} M'_{x_n} \quad \frac{1}{\beta_1} C_2 - \frac{1}{\beta_2} C_4 = \theta_n$$

$$\frac{1}{\beta_2} C_1 - \frac{1}{\beta_1} C_3 = x_n \quad \beta_1 C_2 + \beta_2 C_4 = \frac{1}{EI} V'_{x_n}$$

or:

$$C_1 = \frac{\beta_2^2}{\beta_1^2 + \beta_2^2} \left[\beta_1^2 x_n + \frac{1}{EI} M'_{x_n} \right] \quad (H.38)$$

$$C_2 = \frac{\beta_1^2}{\beta_1^2 + \beta_2^2} \left[-\beta_2^2 x_n + \frac{1}{EI} M'_{x_n} \right] \quad (H.39)$$

$$C_3 = \frac{\beta_1}{\beta_1^2 + \beta_2^2} \left[\beta_2^2 \theta_n + \frac{1}{EI} V'_{x_n} \right] \quad (H.40)$$

$$C_4 = \frac{\beta_2}{\beta_1^2 + \beta_2^2} \left[-\beta_1^2 \theta_n + \frac{1}{EI} V'_{x_n} \right] \quad (H.41)$$

Substituting for C_1 to C_4 into eqs. (H.33) to (H.36) and setting $z=l_n$ yields:

$$x_{n+1} = \frac{1}{\beta_1^2 + \beta_2^2} \left\{ \left[\beta_1^2 \cosh \lambda_1 + \beta_2^2 \cosh \lambda_2 \right] x_n + \left[\beta_1 \sinh \lambda_1 + \beta_2 \sinh \lambda_2 \right] \theta_n + \left[\cosh \lambda_1 - \cosh \lambda_2 \right] \frac{1}{EI} M'_{x_n} \right. \\ \left. + \left[\frac{\beta_2}{\beta_1^2} \sinh \lambda_1 - \frac{\beta_1}{\beta_2^2} \sinh \lambda_2 \right] \frac{1}{EI} V'_{x_n} \right\} \quad (H.42)$$

$$\Theta_{n+1} = \frac{1}{\beta_1^2 + \beta_2^2} \left\{ [\beta_1 \beta_2^2 \sinh \lambda_1 - \beta_1^2 \beta_2 \sinh \lambda_2] x_n + [\beta_2^2 \cosh \lambda_1 + \beta_1^2 \cos \lambda_2] \Theta_n + \left[\frac{\beta_1^2}{\beta_2} \sinh \lambda_1 + \frac{\beta_2^2}{\beta_1} \sin \lambda_2 \right] \frac{1}{EI} M'_{xn} + [\cosh \lambda_1 - \cos \lambda_2] \frac{1}{EI} V'_{xn} \right\} \quad (H.43)$$

$$\frac{1}{EI} M_{x,n+1} = \frac{1}{\beta_1^2 + \beta_2^2} \left\{ \beta_1 \beta_2^2 [\cosh \lambda_1 - \cosh \lambda_2] x_n + \beta_1 \beta_2 [\beta_1 \sinh \lambda_1 - \beta_2 \sinh \lambda_2] \Theta_n + [\beta_2^2 \cosh \lambda_1 + \beta_1^2 \cos \lambda_2] \frac{1}{EI} M'_{xn} + [\beta_1 \sinh \lambda_1 + \beta_2 \sin \lambda_2] \frac{1}{EI} V'_{xn} \right\} \quad (H.44)$$

$$\frac{1}{EI} V_{x,n+1} = \frac{1}{\beta_1^2 + \beta_2^2} \left\{ \beta_1 \beta_2^2 [\beta_1 \sinh \lambda_1 + \beta_2 \sinh \lambda_2] x_n + \beta_1^2 \beta_2^2 [\cosh \lambda_1 - \cosh \lambda_2] \Theta_n + \beta_1 \beta_2 [\beta_2 \sinh \lambda_1 - \beta_1 \sinh \lambda_2] \frac{1}{EI} M'_{xn} + [\beta_2^2 \cosh \lambda_1 + \beta_1^2 \cos \lambda_2] \frac{1}{EI} V'_{xn} \right\} \quad (H.45)$$

These equations can be written:

$$X_{n+1} = a_{1n} X_n + a_{3n} \Theta_n + a_{4n} M'_{xn} + a_{7n} V'_{xn} \\ \Theta_{n+1} = a_{5n} X_n + a_{2n} \Theta_n + a_{6n} M'_{xn} + a_{4n} V'_{xn} \quad (H.46)$$

$$M_{x,n+1} = a_{9n} X_n + a_{10n} \Theta_n + a_{2n} M'_{xn} + a_{3n} V'_{xn}$$

$$V_{x,n+1} = a_{8n} X_n + a_{9n} \Theta_n + a_{5n} M'_{xn} + a_{1n} V'_{xn}$$

Because:

$$\beta_1 \beta_2 = \beta^2$$

the 10 coefficients a_{1n} to a_{10n} become:

$$a_{1n} = \frac{1}{\beta_1^2 + \beta_2^2} [\beta_1^2 \cosh \lambda_1 + \beta_2^2 \cos \lambda_2] \quad (H.47)$$

$$a_{2n} = \frac{1}{\beta_1^2 + \beta_2^2} [\beta_2^2 \cosh \lambda_1 + \beta_1^2 \cos \lambda_2] \quad (H.48)$$

$$a_{3n} = \frac{1}{\beta_1^2 + \beta_2^2} [\beta_1 \sinh \lambda_1 + \beta_2 \sinh \lambda_2] \quad (H.49)$$

$$a_{4n} = \frac{1}{\beta_1^2 + \beta_2^2} [\cosh \lambda_1 - \cos \lambda_2] \frac{1}{EI} \quad (H.50)$$

$$a_{5n} = \frac{\beta^2}{\beta_1^2 + \beta_2^2} [\beta_2 \sinh \lambda_1 - \beta_1 \sinh \lambda_2] \quad (H.51)$$

$$a_{6n} = \frac{1}{\beta^2 (\beta_1^2 + \beta_2^2)} [\beta_2^3 \sinh \lambda_1 + \beta_1^3 \sin \lambda_2] \frac{1}{EI} \quad (H.52)$$

$$a_{7n} = \frac{1}{\beta^4(\beta_1^2 + \beta_2^2)} [\beta_1^3 \sinh \lambda_1 - \beta_2^3 \sin \lambda_2] \frac{1}{EI} \quad (H.53)$$

$$a_{8n} = \nu^2 \rho A a_{3n} \quad (H.54)$$

$$a_{9n} = \nu^2 \rho A EI a_{4n} \quad (H.55)$$

$$a_{10n} = EI a_{5n} \quad (H.56)$$

These 10 coefficients are different for each shaft section since A, I and l_n vary between sections.

In the limit, as λ_1 and λ_2 become very small, the following relationships hold:

$$\begin{aligned} \cosh \lambda_1 &\rightarrow 1 + \frac{1}{2} \beta_1^2 l_n^2 + \frac{1}{24} \beta_1^4 l_n^4 \\ \cos \lambda_2 &\rightarrow 1 - \frac{1}{2} \beta_2^2 l_n + \frac{1}{24} \beta_2^4 l_n^4 \\ \sinh \lambda_1 &\rightarrow \beta_1 l_n [1 + \frac{1}{6} \beta_1^2 l_n^2 + \frac{1}{120} \beta_1^4 l_n^4] \\ \sin \lambda_2 &\rightarrow \beta_2 l_n [1 - \frac{1}{6} \beta_2^2 l_n^2 + \frac{1}{120} \beta_2^4 l_n^4] \end{aligned} \quad (H.57)$$

Hence, in the limit:

$$\begin{aligned} a_{1n} &\rightarrow 1 & a_{2n} &\rightarrow 1 & a_{3n} &\rightarrow l_n \\ a_{4n} &\rightarrow \frac{l_n^2}{2EI} & a_{5n} &\rightarrow \frac{1}{6} \beta_1^4 l_n^3 = \nu^2 \rho A \frac{l_n^3}{6EI} = \frac{1}{6} \frac{\beta_1^4 l_n^4}{l_n} \rightarrow 0 \\ a_{6n} &\rightarrow \frac{l_n}{EI} & a_{7n} &\rightarrow \frac{l_n^3}{6EI} - \frac{l_n}{aAG} & & (H.58) \\ a_{8n} &\rightarrow \nu^2 \rho A l_n & a_{9n} &\rightarrow \frac{1}{2} \nu^2 \rho A l_n^2 & a_{10n} &\rightarrow \frac{1}{2} \nu^2 \rho A l_n^3 \end{aligned}$$

From eq. (H.57) it is seen that these limits are exact when:

$$\frac{1}{24} \beta^4 l_n^4 < 10^{-8} \quad \text{or} \quad \beta l \leq 0.022 \quad (H.59)$$

assuming that the computer works with 8 significant figures. Under these limit conditions and if l_n is not too small, eqs. (H.46) reduce to:

$$\begin{aligned} x_{n+1} &= x_n + l_n \theta_n + \frac{l_n^2}{2EI} M'_{x_n} + \left[\frac{l_n^3}{6EI} - \frac{l_n}{aAG} \right] V'_{x_n} \\ \theta_{n+1} &= \theta_n + \frac{l_n}{EI} M'_{x_n} + \frac{l_n}{2EI} V'_{x_n} \\ M_{x,n+1} &= \frac{1}{2} \nu^2 \rho A l_n^2 x_n + \frac{1}{2} \nu^2 \rho A l_n^2 \theta_n + M'_{x_n} + l_n V'_{x_n} \\ V_{x,n+1} &= \nu^2 \rho A l_n x_n + \frac{1}{2} \nu^2 \rho A l_n^2 \theta_n + V'_{x_n} \end{aligned} \quad (H.60)$$

which are the same expressions as would be obtained if the shaft was considered massless and the actual shaft mass lumped at the endpoints as done in Volume 5.

It is seen that there is no coupling between the x-direction and the y-direction, nor between the cosine and sine-components in eqs. (H.46). Hence, eqs. (H.46) are valid also if the variables are subscripted with c or s, or for the analogous y-components. In this way, eqs. (H.14) to (H.17) together with eqs. (H.46) establish recurrence relationships by which the amplitude, etc. can be calculated step by step, starting from one end of the rotor. Let the rotor end at Station 1 be free, i.e. $M_{x_1} = M_{y_1} = V_{x_1} = V_{y_1} = 0$. First, set $x_1 = 1$ and $y_1 = \theta_1 = \phi_1 = 0$, and use the recurrence formulas to calculate the bending moment and shear force at the last station, station m. Denote the values as: $M'_{x_m} = a_{11}, M'_{y_m} = a_{21}, V'_{x_m} = a_{31}, V'_{y_m} = a_{41}$ (the a's are complex). Next, set $y_1 = 1$ and $x_1 = \theta_1 = \phi_1 = 0$, and calculate $M'_{x_m} = a_{12}, M'_{y_m} = a_{22}, V'_{x_m} = a_{32}, V'_{y_m} = a_{42}$. Repeat the calculations with $\theta_1 = 1$ and $\phi_1 = 1$, respectively. Finally, perform four additional calculations with $x_1 = y_1 = \theta_1 = \phi_1 = 0$, the first with $F_x = 1, F_y = T_x = T_y = 0$, the second with $F_y = 1, F_x = T_x = T_y = 0$, and so on where the forces and moments are applied at that rotor station where the magnetic forces act. Assuming the rotor end at station m to be free, we have $M'_{x_m} = M'_{y_m} = V'_{x_m} = V'_{y_m} = 0$, i.e.:

$$\begin{bmatrix} M'_{x_m} \\ M'_{y_m} \\ V'_{x_m} \\ V'_{y_m} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \\ \phi_1 \end{bmatrix} + \begin{bmatrix} a_{15} & a_{16} & a_{17} & a_{18} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ a_{35} & a_{36} & a_{37} & a_{38} \\ a_{45} & a_{46} & a_{47} & a_{48} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{bmatrix} = 0 \quad (H.61)$$

or:

$$AX_1 = BF \quad (H.62)$$

where:

$$X_1 = \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \\ \phi_1 \end{bmatrix} \quad F = \begin{bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{bmatrix} \quad (H.63)$$

and A and B are matrices defined through the above equations. They are complex such that:

$$A = A_c + iA_s \quad (H.64)$$

$$B = B_c + iB_s$$

and similarly for X, and F. To solve eq. (H.62) for X, write it out into its real and imaginary parts:

$$A_c X_{1c} - A_s X_{1s} = (BF)_c \quad (H.65)$$

$$A_s X_{1c} + A_c X_{1s} = (BF)_s$$

Solve the equations to get:

$$X_{1c} = [A_s^{-1} A_c + A_c^{-1} A_s]^{-1} [A_s^{-1} (B \cdot F)_c + A_c^{-1} (B \cdot F)_s] \quad (H.66)$$

$$X_{1s} = [A_s^{-1} A_c + A_c^{-1} A_s]^{-1} [-A_c^{-1} (B \cdot F)_c + A_s^{-1} (B \cdot F)_s]$$

Define:

$$A' = [A_s^{-1} A_c + A_c^{-1} A_s]^{-1} [A_s^{-1} - iA_c^{-1}] \quad (H.67)$$

whereby the solution of eq. (H.62) becomes:

$$X_1 = A'^{-1} B F \quad (H.68)$$

Let the magnetic forces act at a station with amplitudes:

$$X = \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix} \quad (H.69)$$

In performing the rotor calculations described above, X is expressed by a matrix equation similar to eq. (H.61):

$$X = C X_1 + D F \quad (H.70)$$

Substituting for \mathbf{I}_s from eq. (H.68):

$$\mathbf{X} = [\mathbf{C}\mathbf{A}^{-1}\mathbf{B} + \mathbf{D}] \mathbf{F} = \mathbf{E}^{-1} \mathbf{F} \quad (\text{H.71})$$

where $\mathbf{E}^{-1} = [\mathbf{C}\mathbf{A}^{-1}\mathbf{B} + \mathbf{D}]$ is a complex matrix with 4 rows and 4 columns.

Solve this equation to get:

$$\mathbf{F} = \begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \mathbf{E} \mathbf{X} = \begin{Bmatrix} (\lambda_{11} + i\lambda_{11}) & \dots & (\lambda_{41} + i\lambda_{41}) \\ \vdots & \ddots & \vdots \\ (\lambda_{14} + i\lambda_{14}) & \dots & (\lambda_{44} + i\lambda_{44}) \end{Bmatrix} \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix} \quad (\text{H.72})$$

where \mathbf{E} is obtained as the inverse of \mathbf{E}^{-1} by the same method used to invert \mathbf{A} (see eq. (H.67)). Here:

$$(\lambda_{11} + i\lambda_{11})x = (\lambda_{11} + i\lambda_{11})(x_c + ix_s) = (\lambda_{11}x_c - \lambda_{11}x_s)\cos(\nu t) - (\lambda_{11}x_c + \lambda_{11}x_s)\sin(\nu t) \quad (\text{H.73})$$

and so on. The \mathbf{E} -matrix expresses the impedance of the rotor for the chosen frequency ν at that rotor station where the magnetic forces and moments are applied. It is used in the rotor stability calculation and the rotor response calculation as discussed in Appendices IX and X.

APPENDIX IX: Calculation of the Threshold of Instability for a Rotor with Magnetic Forces

In Appendix VIII it is shown that the rotor can be represented by an impedance matrix E at the point where the magnetic forces and moments are applied. This matrix depends on the vibratory frequency ν and relates the rotor amplitudes and slopes to the imposed forces (see eq. (H.72), Appendix VIII). The magnetic forces F_x and F_y and moments, T_x and T_y , on the other hand, also depend on the rotor amplitudes x and y and slopes θ and ϕ , and can be written:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = - \left[\begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\phi} \\ Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\phi} \\ Q_{ox} & Q_{oy} & Q_{o\theta} & Q_{o\phi} \\ Q_{px} & Q_{py} & Q_{p\theta} & Q_{p\phi} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\theta} & q_{x\phi} \\ q_{yx} & q_{yy} & q_{y\theta} & q_{y\phi} \\ q_{ox} & q_{oy} & q_{o\theta} & q_{o\phi} \\ q_{px} & q_{py} & q_{p\theta} & q_{p\phi} \end{Bmatrix} \sin(\Omega t) \right] \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix} \quad (J.1)$$

where Ω radians/sec is the frequency of the magnetic forces, and the Q 's and q 's are the gradients of the magnetic forces and moments. In most cases, several of the gradients are zero and there also exist certain symmetry relationships between the gradients. However, all the terms will be kept in the analysis to make it general.

Combining eqs. (J.1) and eq. (H.72), Appendix VIII, a matrix equation is obtained with the rotor amplitudes x and y and the rotor slopes θ and ϕ as the unknowns.

To solve the equation it is necessary to expand x , y , θ and ϕ in Fourier series as:

$$x = \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] \quad (J.2)$$

$$y = \sum_{k=0}^{\infty} [y_{ck} \cos(k\psi) - y_{sk} \sin(k\psi)] \quad (J.3)$$

and similarly for θ and ϕ where:

$$\psi = \frac{1}{2} \Omega t \quad (J.4)$$

Thus, for a given k the frequency is:

$$\nu = k \frac{Q}{2} \quad (J.5)$$

and the elements of the impedance matrix E (i.e. the X 's and λ 's of eq. (H.72), Appendix VII) are evaluated at these frequencies such that there will be an E -matrix for each value of k :

$$E_k = E_{Ck} + i E_{Sk} \quad (J.6)$$

Next, define:

$$X_k = \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix}_k = X_{Ck} + i X_{Sk} \quad (J.7)$$

where:

$$X_{Ck} = \begin{Bmatrix} x_{Ck} \\ y_{Ck} \\ \theta_{Ck} \\ \phi_{Ck} \end{Bmatrix} \quad X_{Sk} = \begin{Bmatrix} x_{Sk} \\ y_{Sk} \\ \theta_{Sk} \\ \phi_{Sk} \end{Bmatrix} \quad (J.8)$$

Also, set:

$$Q = \begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{x\theta} & Q_{x\phi} \\ Q_{yx} & Q_{yy} & Q_{y\theta} & Q_{y\phi} \\ Q_{x\theta} & Q_{y\theta} & Q_{\theta\theta} & Q_{\theta\phi} \\ Q_{x\phi} & Q_{y\phi} & Q_{\theta\phi} & Q_{\phi\phi} \end{Bmatrix} \quad (J.9)$$

$$q = \begin{Bmatrix} q_{xx} & q_{xy} & q_{x\theta} & q_{x\phi} \\ q_{yx} & q_{yy} & q_{y\theta} & q_{y\phi} \\ q_{x\theta} & q_{y\theta} & q_{\theta\theta} & q_{\theta\phi} \\ q_{x\phi} & q_{y\phi} & q_{\theta\phi} & q_{\phi\phi} \end{Bmatrix} \quad (J.10)$$

With these definitions, eqs. (H.72) and (J.1) can be combined to yield:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = \sum_{k=0}^{\infty} E_k X_k [\cos(k\psi) + i \sin(k\psi)] = -[Q \cos(2\psi) - q \sin(2\psi)] \sum_{k=0}^{\infty} X_k [\cos(k\psi) - \sin(k\psi)] \quad (J.11)$$

or in expanded form:

$$\begin{aligned} & \sum_{k=0}^{\infty} [(E_{ck} X_{ck} - E_{sk} X_{sk}) \cos(k\psi) - (E_{sk} X_{ck} + E_{ck} X_{sk}) \sin(k\psi)] \\ & + \frac{1}{2} Q \sum_{k=0}^{\infty} X_{ck} [\cos(k+2)\psi + \cos(k-2)\psi] - \frac{1}{2} q \sum_{k=0}^{\infty} X_{sk} [\cos(k+2)\psi - \cos(k-2)\psi] \\ & - \frac{1}{2} q \sum_{k=0}^{\infty} X_{ck} [\sin(k+2)\psi - \sin(k-2)\psi] - \frac{1}{2} Q \sum_{k=0}^{\infty} X_{sk} [\sin(k+2)\psi + \sin(k-2)\psi] = 0 \end{aligned} \quad (J.12)$$

By collecting terms in $\cos(k\psi)$ and $\sin(k\psi)$, two sets of equations are obtained, one set for k even ($k=0, 2, 4, \dots$) and one set for k odd ($k=1, 3, 5, \dots$).

Consider first the case of k even. When $k \geq 4$, eq. (J.12) yields for any k :

k even, $k \geq 4$

$$E_{ck} X_{ck} - E_{sk} X_{sk} + \frac{1}{2} Q X_{ck,k-2} - \frac{1}{2} q X_{sk,k-2} + \frac{1}{2} Q X_{ck,k+2} + \frac{1}{2} q X_{sk,k+2} = 0 \quad (J.13)$$

$$E_{sk} X_{ck} + E_{ck} X_{sk} + \frac{1}{2} q X_{ck,k-2} + \frac{1}{2} Q X_{sk,k-2} - \frac{1}{2} q X_{ck,k+2} + \frac{1}{2} Q X_{sk,k+2} = 0$$

which can be written:

$$(E_{ck} + iE_{sk})(X_{ck} + iX_{sk}) + \frac{1}{2}(Q+iq)(X_{ck,k-2} + iX_{sk,k-2}) + \frac{1}{2}(Q-iq)(X_{ck,k+2} + iX_{sk,k+2}) = 0 \quad (J.14)$$

or:

$$G X_{k-2} + E_k X_k + H X_{k+2} = 0 \quad (J.15)$$

where:

$$G = \frac{1}{2}(Q+iq) \quad H = \frac{1}{2}(Q-iq) \quad (J.16)$$

Define the matrix S_k by:

$$X_k = S_{k-2} X_{k-2} \quad (J.17)$$

Substitute into eq. (J.15) to get:

$$G X_{k-2} + [E_k + H S_k] X_k = 0 \quad (J.18)$$

or:

$$X_k = -[E_k + H S_k]^{-1} G X_{k-2} \quad k \geq 4 \quad (J.19)$$

By comparing eqs. (J.19) and (J.17):

$$S_{k-2} = -[E_k + HS_k]^{-1}G \quad k \geq 1 \quad (J.20)$$

For $k=2$, eq. (J.12) yields:

$$2GX_{c_0} + E_2 X_2 + HX_4 = 0 \quad (J.21)$$

since $X_{s_0} = 0$. Hence:

$$X_2 = -[E_2 + HS_2]^{-1}2GX_{c_0} \quad (J.22)$$

Thus, eq. (J.17) is valid also for $k=2$ if it is defined that:

$$S_0 = -[E_2 + HS_2]^{-1}2G \quad (J.23)$$

Turning last to the case of $k=0$, eq. (J.12) yields:

$$E_{c_0} X_{c_0} + \frac{1}{2}QX_{c_2} + \frac{1}{2}qX_{s_2} = 0 \quad (J.24)$$

or by introducing eq. (J.22):

$$[E_{c_0} + \frac{1}{2}QS_{c_0} + \frac{1}{2}qS_{s_0}]X_{c_0} = 0 \quad (J.25)$$

where:

$$S_{c_0} + iS_{s_0} = S_0 \quad (J.26)$$

The coefficient matrix on the left hand side of eq. (J.25) is a 4 by 4 real matrix. In order for a non-trivial solution of X_{c_0} to exist it is necessary that the determinant is zero, and in that case the rotor is unstable.

Turning next to the case of k odd and taking $k \geq 3$, eq. (J.12) yields for a given value of k , equations identical to eq. (J.15). Hence eqs. (J.17) to (J.20) are

also valid for $k \geq 3$. However, for $k=1$ eq. (J.12) gives:

$$(E_{c1} + \frac{1}{2}Q)X_{c1} - (E_{s1} - \frac{1}{2}q)X_{s1} + \frac{1}{2}QX_{c3} + \frac{1}{2}qX_{s3} = 0 \quad (J.27)$$

$$(E_{s1} + \frac{1}{2}q)X_{c1} + (E_{c1} - \frac{1}{2}Q)X_{s1} - \frac{1}{2}qX_{c3} + \frac{1}{2}QX_{s3} = 0$$

By substitution from eq. (J.17), eq. (J.27) becomes:

$$[E_1 + HS_1]X_1 + \frac{1}{2}QX_{c1} + \frac{1}{2}qX_{s1} + i(\frac{1}{2}qX_{c1} - \frac{1}{2}QX_{s1}) = 0 \quad (J.28)$$

Equating real and imaginary parts, this equation yields an 8 by 8 real matrix whose determinant must vanish at the threshold of instability of the rotor.

In order to evaluate the two determinants, the one for even values of k from eq. (J.25) and the one for odd values of k from eq. (J.28), it is necessary to calculate S_0 and S_1 . This is done by using the recurrence relationship of eq. (J.20) where S_{k-2} can be found when S_k is known (for $k=2$, use eq. (J.23)). Now, the elements of the impedance matrix E_k are of the order k^2 (except if $k \frac{\Omega}{2}$ is close to a resonant frequency of the system). Thus, for sufficiently high values of k , S_k will be of the order k^{-2} such that it is possible to ignore all S_k -matrices for $k \geq p$ where p is selected on the basis of the desired accuracy of the calculations. With $S_p = S_{p+1} = 0$, eq. (J.20) yields:

$$S_{p-1} = -E_{p+1}^{-1} G \quad (J.29)$$

$$S_{p-2} = -E_p^{-1} G$$

after which eq. (J.20) and eq. (J.23) can be employed to calculate S_k , $p-3 \leq k \leq 0$, keeping the S -matrices for even values of k separate from the S -matrices for odd values of k . Once S_0 and S_1 have been obtained the two determinants can be calculated.

To perform a complete stability analysis of a rotor, the rotor dimensions, the rotor speed and the bearing coefficients must be specified. It is then possible to calculate the impedance matrices of the rotor (the E_k -matrices), by the method explained in Appendix VI. Next, to determine if the rotor is stable or unstable,

assume the magnetic force gradients (i.e. the Q's and q's of eq. (J.1)) to be variable but such that their mutual ratio is kept constant and equal to their specified value. In other words, introduce a reference value, Q_{ref} , (for instance, $Q_{ref} = Q_{xx}$) and let Q_{ref} be the single variable but such that when Q_{ref} varies, the ratios Q_{yy}/Q_{ref} , q_{yy}/Q_{ref} , and so on remain fixed. Then increase Q_{ref} in steps, starting with $Q_{ref}=0$, and calculate the corresponding values of the two determinants as discussed above. In this way the determinants are obtained as functions of Q_{ref} . If neither of the determinants become zero between $Q_{ref}=0$ and that value of Q_{ref} where the Q's and q's assume their specified values, the rotor is stable, otherwise unstable. It should be noted that this assumes the rotor to be stable at $Q_{ref}=0$, i.e. when there are no magnetic forces. Even if the two determinants are not zero for $Q_{ref}=0$, the rotor may still be unstable with hydrodynamic whirl instability induced by the fluid film forces in the bearings. This latter form of instability cannot be analyzed by the present method but must be checked by the methods given in Volume 5.

APPENDIX X: Calculation of the Amplitude Response of a Rotor with Magnetic Forces

When there is a built-in eccentricity between the rotor center and the magnetic axis of the generator stator, the magnetic forces will force the rotor to whirl. Let this built-in eccentricity be described by $(x_0, y_0, \theta_0, \phi_0)$ where x_0 and y_0 give the coordinates of the rotor center with respect to the stator center and θ_0 and ϕ_0 give the angles between the rotor axis and the stator axis. These values include the contributions from the static components of the magnetic forces. Then the magnetic forces F_x and F_y and the magnetic moments T_x and T_y can be expressed in terms of the rotor amplitudes x and y , the rotor slopes θ and ϕ , and x_0, y_0, θ_0 and ϕ_0 :

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = -[Q \cos(\Omega t) - q \sin(\Omega t)] \begin{Bmatrix} x_0 + x \\ y_0 + y \\ \theta_0 + \theta \\ \phi_0 + \phi \end{Bmatrix} \quad (K.1)$$

where Q and q are matrices defined by eqs. (J.9) and (J.10), Appendix IX. These forces and moments act on the rotor where they produce the amplitudes x and y and the slopes θ and ϕ which are related to the forces through the impedance matrix E as given by eq. (H.72), Appendix VIII. Thus, by combining eq. (H.72) and eq. (K.1) a matrix equation is obtained with x, y, θ and ϕ as the unknowns. To solve this equation, expand x, y, θ and ϕ in Fourier series:

$$x = \sum_{k=0}^{\infty} [x_{ck} \cos(k\psi) - x_{sk} \sin(k\psi)] \quad (K.2)$$

and similarly for y, θ and ϕ where:

$$\psi = \Omega t \quad (K.3)$$

Thus, for a particular value of k the frequency is:

$$\nu = k\Omega \quad (K.4)$$

It should be noted that this differs from the stability analyses where $\psi = \frac{1}{2}\Omega t$ and $\nu = k \frac{\Omega}{2}$. In the response calculation only multiples of the magnetic force frequency are considered or, in terms of the stability analysis, only the case of

"k even" is considered.

For each k-value a rotor impedance matrix E_k can be computed as shown in Appendix VIII

$$E_k = E_{ck} + iE_{sk} \quad (K.5)$$

Define:

$$X_k = \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix}_k = X_{ck} + iX_{sk} \quad (K.6)$$

where:

$$X_{ck} = \begin{Bmatrix} x_{ck} \\ y_{ck} \\ \theta_{ck} \\ \phi_{ck} \end{Bmatrix} \quad X_{sk} = \begin{Bmatrix} x_{sk} \\ y_{sk} \\ \theta_{sk} \\ \phi_{sk} \end{Bmatrix} \quad (K.7)$$

Then combine eqs. (H.72) and (K.1) to get:

$$\sum_{k=0}^{\infty} E_k X_k [\cos(k\psi) + i\sin(k\psi)] = -[Q\cos(\Omega t) - q\sin(\Omega t)] \left[\begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix} + \sum_{k=0}^{\infty} X_k [\cos(k\psi) + i\sin(k\psi)] \right] \quad (K.8)$$

which can be expanded into the form:

$$\begin{aligned} & \sum_{k=0}^{\infty} [(E_{ck} X_{ck} - E_{sk} X_{sk}) \cos(k\psi) - (E_{sk} X_{ck} + E_{ck} X_{sk}) \sin(k\psi)] \\ & + \frac{1}{2} Q \sum_{k=0}^{\infty} X_{ck} [\cos(k+1)\psi + \cos(k-1)\psi] - \frac{1}{2} q \sum_{k=0}^{\infty} X_{sk} [\cos(k+1)\psi - \cos(k-1)\psi] \\ & - \frac{1}{2} q \sum_{k=0}^{\infty} X_{ck} [\sin(k+1)\psi - \sin(k-1)\psi] - \frac{1}{2} Q \sum_{k=0}^{\infty} X_{sk} [\sin(k+1)\psi + \sin(k-1)\psi] \end{aligned} \quad (K.9)$$

$$= -(Q\cos\psi - q\sin\psi) \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix}$$

Collecting terms in $\cos(k\psi)$ and $\sin(k\psi)$, this equation gives rise to an infinite number of simultaneous equations. For $k \geq 2$ and for any arbitrary value of k , the equations become:

$$\begin{aligned} & \sum_{k=2}^{\infty} E_{ck} X_{ck} - E_{sk} X_{sk} + \frac{1}{2} Q X_{c,k-1} - \frac{1}{2} q X_{s,k-1} + \frac{1}{2} Q X_{c,k+1} + \frac{1}{2} q X_{s,k+1} = 0 \\ & E_{sk} X_{ck} + E_{ck} X_{sk} + \frac{1}{2} q X_{c,k-1} + \frac{1}{2} Q X_{s,k-1} - \frac{1}{2} q X_{c,k+1} + \frac{1}{2} Q X_{s,k+1} = 0 \end{aligned} \quad (K.10)$$

or in complex notation:

$$\underline{k \geq 2} \quad GX_{k-1} + EX_k + HX_{k+1} = 0 \quad (K.11)$$

where the G and H matrices are defined by eq. (J.16), Appendix IX. For $k=1$, the right hand side is not zero. The equations become:

$$\underline{k=1} \quad E_{C1}X_{C1} - E_{S1}X_{S1} + QX_{C0} + \frac{1}{2}QX_{C2} + \frac{1}{2}qX_{S2} = -Q \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \\ Q_0 \end{Bmatrix} \quad (K.12)$$

$$E_{S1}X_{C1} + E_{C1}X_{S1} + qX_{C0} - \frac{1}{2}qX_{C2} + \frac{1}{2}QX_{S2} = -q \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \\ Q_0 \end{Bmatrix}$$

or in complex notation:

$$2GX_{C0} + EX_1 + HX_2 = -2G \begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \\ Q_0 \end{Bmatrix} \quad (K.13)$$

since $X_{S0} = 0$.

Finally, for $k=0$ eq. (K.9) yields:

$$E_{C0}X_{C0} + \frac{1}{2}QX_{C1} + \frac{1}{2}qX_{S1} = 0 \quad (K.14)$$

Now, define a matrix S_k by:

$$\underline{k \geq 2} \quad X_k = S_{k-1}X_{k-1} \quad (K.15)$$

and substitute into eq. (K.11) to get:

$$GX_{k-1} + (E_k + HS_k)X_k = 0$$

or:

$$X_k = -[E_k + HS_k]^{-1}GX_{k-1} \quad (K.16)$$

Hence:

$$S_{k-1} = -[E_k + HS_k]^{-1} G \quad k \geq p \quad (K.17)$$

As previously discussed, E_k is of the order k^2 which means that for sufficiently large values of k , S_k is of the order k^{-2} . Hence, for $k \geq p$ where the choice of p depends on the desired accuracy of the calculation, S_k may be set equal to zero, i.e.:

$$S_{p-1} = -E_p^{-1} G \quad (K.18)$$

Thereafter eq. (K.17) can be used to calculate all subsequent S_k matrices, $p-2 \geq k \geq 1$. Having obtained S_1 , eq. (K.12) becomes:

$$2G X_{c_0} + [E_1 + HS_1] X_1 = -2G \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix} \quad (K.19)$$

with the solution:

$$X_1 = -[E_1 + HS_1]^{-1} 2G [X_{c_0} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix}] = 2S_0 [X_{c_0} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix}] \quad (K.20)$$

where:

$$S_0 = S_{c_0} + i S_{s_0} = -[E_1 + HS_1]^{-1} G \quad (K.21)$$

With this result, eq. (K.14) can be written:

$$[E_{c_0} + QS_{c_0} + q S_{s_0}] X_{c_0} = -[QS_{c_0} + q S_{s_0}] \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix} \quad (K.22)$$

or:

$$[E_{c_0} + QS_{c_0} + q S_{s_0}] [X_{c_0} + \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix}] = E_{c_0} \begin{Bmatrix} x_0 \\ y_0 \\ \theta_0 \\ \phi_0 \end{Bmatrix} \quad (K.23)$$

These four equations may be solved for X_{c_0} or $[X_{c_0} + \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}]$ after which X_k can be calculated from eq. (K.20) and all other X_k -vectors from eq. (K.15). Thereby the complete amplitude response is obtained at the station where the magnetic forces are applied. To determine the response at other stations, the rotor calculation has resulted in relationships:

$$\text{at station } n \quad X_{nk} = C_{hk} X_{1k} + D_{hk} F_k \quad (\text{K.24})$$

Here X_{1k} is related to F_k by eq. (H.63):

$$X_{1k} = A_k^{-1} B_k F_k \quad (\text{K.25})$$

i.e.:

$$X_{hk} = [C_{hk} A_k^{-1} B_k + D_{hk}] F_k \quad (\text{K.26})$$

or with F_k given by eq. (E.72):

$$X_{hk} = [C_{hk} A_k^{-1} B_k + D_{hk}] E_k X_k \quad (\text{K.27})$$

Thus, with X_k determined, the amplitudes and slopes at any other rotor station can be found.

APPENDIX XI: Computer Program - The Stability of a Rotor with Timevarying Magnetic Forces

This appendix describes the computer program PN0351: "The Stability of a Rotor with Timevarying Magnetic Forces" and gives the detailed instructions for using the program. The program is based on the analysis contained in Appendix IX (and Appendix VIII). It calculates that value of the gradient of the timevarying magnetic force which is required to make the rotor unstable.

The rotor-bearing model is that of a general flexible rotor supported in a number of bearings (see Fig. 4). The dynamic bearing reactions are represented by 4 spring coefficients and 4 damping coefficients (see Volume 3). The rotor itself consists of a shaft whose diameter may vary in steps along the rotor and on which are fastened any number of masses (wheels, impellers, collars, etc.). At the centerplane of the alternator stator, timevarying magnetic forces act on the rotor. These forces may be both forces and moments and they are directly proportional to the rotor amplitudes and slopes. The forces depend strongly on the type of alternator and can be determined as discussed elsewhere in this report.

The stability computer program is not "automatic" in the sense that all that is required is to provide the numerical data describing the rotor and the magnetic forces, and then the program will calculate if the rotor is stable or not. It must definitely be emphasized that it requires judgement and several calculations to determine the rotor stability. A more detailed discussion of the problem is given in the text of the report and it is necessary to read that before attempting to use the program.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when preparing the computer input. In the following the more detailed instructions are given.

Card 1 (72H) Any descriptive text may be given, identifying the calculation.

Card 2 (12I5) This is the "control card" whose values control the rest of the input. Thus, in order to understand some of the items it is necessary to refer to that particular part of the data to which the control number apply.

1.NS, specifies the number of rotor stations, see "Rotor Data" ($NS \leq 100$).

2.NB, specifies the number of bearings ($1 \leq NB \leq 10$)

3.KA, The absolute value of KA, $|KA|$, specifies the number of that rotor station at which the magnetic forces are applied (i.e. at the centerplane of the alternator).

The program only provides for one such station.

When $KA > 0$ and $KC \geq 0$ (see next item), there are only timevarying magnetic forces and no moments. When $KA < 0$ and $KC \geq 0$, there are only timevarying magnetic moments and no forces. When $KC \leq -1$, $KA > 0$ there are both timevarying magnetic forces and moments. For further details, see "Magnetic Force data."

4.KC KC is used to specify the form of the timevarying magnetic forces. When $KC=0$, the timevarying magnetic forces ($KA > 0$) or moments ($KA < 0$) depend only on the rotor amplitudes, not on the rotor slopes. When $KC=1$, the timevarying magnetic forces ($KA > 0$) or moments ($KA < 0$) depend only on the rotor slopes, not the rotor amplitudes. Finally, when $KC=-1$ there are both timevarying magnetic forces and moments, and they depend both on the rotor amplitudes and the rotor slopes. The reason for including this control parameter is to reduce the size of the stability determinant whenever possible. For further details, see "Magnetic Force Data."

5.NRP The program provides for including the effect of the bearing support pedestals whenever needed. If $NRP=0$, the bearing pedestals are rigid and no pedestal data is required in the input. Otherwise, set $NRP=1$ and specify the pedestal data.

For further details, see "Pedestal Data."

6.NPD The bearings supporting the rotor may be either of the fixed geometry type (plain cylindrical bearings, grooved sleeve bearings, ball bearings, and so on) or they may be tilting pad bearings. If the bearings are of the fixed geometry type, set NPD=0. If tilting pad bearings are employed, the absolute value of NPD, $|NPD|$, specifies the number of pads in the bearing. The bearing may be oriented such that the static bearing reaction goes between the two bottom pads in which case NPD is positive and equal to the number of pads. However, if the bearing is oriented such that the static bearing reaction passes through the pivot of the bottom pad, NPD is negative and equal to minus the number of bearing pads. The maximum value of $|NPD|$ is 8. For further details, see "Bearing Data, Tilting Pad Bearing."

7.INC. The bearing lubricant may be incompressible (INC=0) as for oil bearings or it may be compressible (INC=1) as for gas bearings. The difference in so far as the program is concerned, is that the dynamic bearing coefficients with a compressible lubricant are functions of the vibratory frequency which they are not when the lubricant is incompressible. For further details, see "Bearing Data."

8.NH For the stability calculation, the program evaluates the frequency response of the rotor-bearing system. Theoretically, infinitely many frequencies are required, but in practice only a limited number are necessary. These frequencies are the half harmonics of the magnetic force frequency and NH specifies the number of the highest half harmonic. NH must be equal to or greater than 2 but cannot exceed 20. NH should not be made greater than necessary since the computer time is almost proportional to the value of NH, and in many practical cases sufficient accuracy is obtained by setting NH=2. A more detailed discussion is given in the text. See also "Bearing Data."

9.NO Each stability calculation is performed at a given rotor speed. With the speed fixed the program varies the gradients of the timevarying magnetic forces over a specified range to determine when instability is encountered. The variable

representing the magnetic force gradients is called Q_{ref} . If $NQ=0$, a range for Q_{ref} is employed, and it is necessary to specify the first and the last value of the range and also the increment by which the range should be covered. If $NQ \geq 1$, a list of NQ - values of Q_{ref} is given. For further details, see "Test Range of Magnetic Force Gradients."

10. NSP When it is desired to investigate several speed ranges for the same rotor and where either the bearing coefficients or the magnetic forces change from speed range to speed range, it is convenient not to have to repeat the "fixed" rotor data for each calculation. NSP gives the number of such speed ranges. The input data, starting from "Speed Data," must be repeated NSP times. NSP can be any positive value desired.

11. NDIA If NDIA=0, the program output will be limited to giving the value of the two stability determinants as a function of Q_{ref} for each rotor speed. In general, this should be adequate. However, in some cases it may be desired to study the behavior of the rotor in some detail. By setting NDIA=1, the computer output will also include the impedance matrices of the rotor at each of the specified frequencies and it can be investigated if any of the harmonics coincide with a resonance of the system. If NDIA=-1, the output will include not only the rotor impedance matrices but also the S_k -matrices employed in solving the determinants. (See Appendix IX).

12. INP If INP=0, the computer will expect to read in a completely new set of input data, starting from card 1, upon finishing the calculations for the present input data. Otherwise, set INP=1.

Card 3 (1P5E14.6)

1.YM YM gives Youngs modulus E for the shaft material in lbs/inch². If E actually changes along the rotor, it should be noted that the program only uses E in the produce EI where I is the cross-sectional moment of inertia of the shaft. Since $I = \frac{\pi}{64} (d_o^4 - d_i^4)$, where d_o is the outer shaft diameter and d_i is the inner shaft diameter, any variation in E can be absorbed by changing d_o (see "Rotor Data").

2. DNST specifies the weight density of the shaft material, lbs/inch³. The program converts it into the mass density $\rho = \text{DNST}/386.069$.

3.SHM gives the product αG where G is the shear modulus, lbs/inch², and α is the shape factor for shear (for circular cross-sections: $\alpha \approx 0.75$)

Rotor Data (8E9.2.)

Referring to figure 4, the rotor is represented by a number of stations connected by shaft sections of uniform diameter. Thus, rotor stations are introduced wherever the shaft diameter changes (or changes significantly). Also, there must be a rotor station at each end of the rotor, at each bearing centerline and at the centerplane of the alternator where the magnetic forces are applied (Station KA).

Furthermore, a rotor station is introduced wherever the shaft has a concentrated mass which cannot readily be represented in terms of an inner and outer shaft diameter (impellers, turbine wheels, alternator poles, and so on). In this way the rotor is assigned a total of NS stations (card 2, item 1) which are numbered consecutively starting from one end of the rotor. There can be a maximum of 100 stations. Each station can be assigned a concentrated mass m with a polar mass moment of inertia I_p and a transverse mass moment of inertia I_T (any of these quantities may, of course, be zero). Also, each station can be assigned a shaft section with which it is connected to the following station. This shaft section has a length l , an outer diameter $(d_o)_{\text{stiff}}$, an outer diameter $(d_o)_{\text{mass}}$ and an inner diameter d_i . The outer diameter $(d_o)_{\text{stiff}}$ is used to specify the stiffness of the shaft section such that the cross-sectional moment of inertia of the shaft is: $I = \frac{\pi}{64} [(d_o)_{\text{stiff}}^4 - d_i^4]$ and the shear area is: $A = \frac{\pi}{4} [(d_o)_{\text{stiff}}^2 - d_i^2]$. The outer diameter $(d_o)_{\text{mass}}$ is used in calculating the mass of the shaft such that the mass per unit length is: $\rho \cdot \frac{\pi}{4} [(d_o)_{\text{mass}}^2 - d_i^2]$ where ρ is the mass density (see card 3, item 2).

In the computer input there must be a card for each rotor station (NS cards). Each card specifies the 7 values for the station:

1. The concentrated mass: m , lbs. (may be zero)
2. The polar mass moment of inertia of the station mass; I_p lbs-inch² (may be zero).
3. The transverse mass moment of inertia of the station mass; I_t lbs-inch² (may be zero).
4. The length of the shaft section to the next station: ℓ , inch. (may be zero).
For the last station, set $\ell = 0$.
5. The outer diameter, $(d_o)_{stiff}$ of the shaft section, inch. $(d_o)_{stiff}$ is used in calculating the stiffness of the shaft section; $(d_o)_{stiff} \neq 0$.
For the last station, set $(d_o)_{stiff} = 1.0$.
6. The outer diameter, $(d_o)_{mass}$ of the shaft section, inch. (may be zero).
 $(d_o)_{mass}$ is used in calculating the mass of the shaft section. For the last station, set $(d_o)_{mass} = 0$.
7. The inner diameter, d_i of the shaft section, inch (may be zero). d_i is used both in calculating the stiffness and the mass of the shaft section.
For the last station, set $d_i = 0$.

Bearing Stations (1215)

The rotor bearing station numbers at which there are bearings, are listed in sequence. There can be up to 10 bearings.

Pedestal Data (8E9.2)

The program provides for the option that the pedestals supporting the bearings may be flexible. In that case, data for the pedestals must be given and NRP must be set equal to 1 (card 2, item 5). If the pedestals are rigid, set NRP=0 and omit giving any data for the pedestals.

When NRP=1, each bearing is supported in a "two-dimensional" pedestal. The pedestal is represented as two separate masses, each mass on its own spring and dashpot. The one mass-spring-dashpot system represents the pedestal characteristics in the x-direction, (the vertical direction) and the other system represents the y-direction (the horizontal direction). There is no coupling between the two systems. In the computer input there must be one card for each rotor bearing which gives the 6 items necessary to specify the pedestal characteristics:

1. The pedestal mass for the x-direction, lbs.
2. The pedestal stiffness for the x-direction, lbs/inch
3. The pedestal damping coefficient for the x-direction, lbs-sec/inch
(note: for the bearing films the damping is given in lbs/inch, whereas the damping coefficient in lbs-sec/inch is used for the pedestals)
4. The pedestal mass for the y-direction, lbs.
5. The pedestal stiffness for the y-direction, lbs/inch
6. The pedestal damping coefficient for the y-direction, lbs-sec/inch.

Speed Data (1PSE14.6)

Usually it is desired to make calculations not just for a single rotor speed but for a range of speeds. Even though the bearing coefficients and also the magnetic forces are somewhat dependent on speed, it is convenient to be able to perform calculations for several speeds without having to change the bearing coefficient data and the magnetic force data. The present input card allows specifying such a speed range by giving the first speed and the last speed of the range and the increment by which the range should be covered. If it is desired to run only one speed, let the initial speed be equal to the desired speed and let the final speed be less than this value whereas the speed increment is set equal to zero. The speed data card also specifies the ratio between the frequency of the timevarying magnetic forces and the speed of the rotor. For the 4 pole homopolar generator, this ratio is equal to 2, and for the heteropolar generator under load, the ratio is equal

to the number of rotor teeth. Finally, the speed data card also specifies the scalefactor for the stability determinants. Hence, in total the speed data card has five values:

1. Initial speed of speed range, rpm
2. Final speed of speed range, rpm
3. Increment by which the speed range is covered, rpm
4. Ratio between magnetic force frequency and rotor speed (=2 for the 4 pole homopolar generator, and equal to the number of rotor teeth for the heteropolar generator under load)
5. Scalefactor for the stability determinants. In order to control computer overflow, each element of the two stability determinants (i.e. for even and odd indicies) is divided by the product of the specified scalefactor and the square of the angular rotor speed. It is recommended to set the scalefactor equal to the rotor mass (in lbs-sec²/in) times approximately 1/8 the square of the product of the highest harmonic and the magnetic force frequency ratio. However, the choice of scalefactor in no way influences the accuracy of the calculations and if in doubt, set the scalefactor equal to 1. If overflow is encountered, increase the scalefactor.

Magnetic Force Data

The generator magnetic forces are made up of two parts: a static component and a timevarying component. Let the rotor amplitudes at the centerplane of the alternator be x and y , and let the corresponding slopes of the rotor be Θ and Φ (i.e. $\Theta = \frac{dx}{dz}$, $\Phi = \frac{dy}{dz}$ where z is the coordinate along the rotor). Then the magnetic forces and moments acting on the rotor are proportional to x, y, Θ and Φ . The static components can be written:

$$\begin{array}{ll} \text{static magnetic forces} & \left\{ \begin{array}{l} F_x = Q_0 x \\ F_y = Q_0 y \end{array} \right. \\ \text{static magnetic moments} & \left\{ \begin{array}{l} T_x = Q'_0 \Theta \\ T_y = Q'_0 \Phi \end{array} \right. \end{array}$$

where Q_0 is the negative radial stiffness in lbs/inch and Q'_0 is the negative angular stiffness in lbs- inch/radian.

The first card of the magnetic force data specify the two negative static stiffnesses.

(1P5E14.6)

1. The negative of the radial static stiffness Q_0 , lbs/inch
2. The negative of the angular static stiffness Q'_0 , lbs-inch/radian

Q_0 and Q'_0 are positive values, and the program assumes them to act as negative springs. For the heteropolar generator, $Q'_0 = 0$.

This card is followed by several cards specifying the gradients of the time-varying magnetic forces and moments. These forces and moments are written:

$$\begin{Bmatrix} F_x \\ F_y \\ T_x \\ T_y \end{Bmatrix} = -Q_{ref} \begin{Bmatrix} Q_{xx} & Q_{xy} & Q_{xe} & Q_{x\varphi} \\ Q_{yx} & Q_{yy} & Q_{ye} & Q_{y\varphi} \\ Q_{xe} & Q_{ye} & Q_{ee} & Q_{e\varphi} \\ Q_{x\varphi} & Q_{y\varphi} & Q_{e\varphi} & Q_{pp} \end{Bmatrix} \cos(\Omega t) - \begin{Bmatrix} q_{xx} & q_{xy} & q_{xe} & q_{x\varphi} \\ q_{yx} & q_{yy} & q_{ye} & q_{y\varphi} \\ q_{xe} & q_{ye} & q_{ee} & q_{e\varphi} \\ q_{x\varphi} & q_{y\varphi} & q_{e\varphi} & q_{pp} \end{Bmatrix} \sin(\Omega t) \begin{Bmatrix} x \\ y \\ \theta \\ \varphi \end{Bmatrix} \quad (L.1)$$

The units of the gradients are:

$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}$; $q_{xx}, q_{xy}, q_{yx}, q_{yy}$ in lbs/inch

$Q_{xe}, Q_{x\varphi}, Q_{ye}, Q_{y\varphi}, q_{xe}, q_{x\varphi}, q_{ye}, q_{y\varphi}$ in lbs/radian

$Q_{ee}, Q_{e\varphi}, Q_{pp}, Q_{pp}, q_{ee}, q_{e\varphi}, q_{pp}, q_{pp}$ in lbs-inch/inch

$Q_{ee}, Q_{e\varphi}, Q_{pp}, Q_{pp}, q_{ee}, q_{e\varphi}, q_{pp}, q_{pp}$ in lbs-inch/radian

The program searches for the threshold of instability by varying a reference value Q_{ref} , representing the gradients of the timevarying magnetic forces (see the above equation), but the matrices themselves remain unchanged in the search operation. Thus, when $Q_{ref}=1$, the actual operating conditions of the alternator is encountered (or, in general, when $Q_{ref}'(Q_{xx})_{input}$, etc. is

equal to the actual value.)

The program provides for five possibilities: either give the full matrices as shown, or any of the four submatrices. Which option to use depends on the generator type, and it is specified by means of the two control numbers on card : KC (item 4) and the sign of KA (item 3).

For the 4 pole homopolar generator or the two-coil Lundell generator where the north poles and the southpoles are in separate planes, the complete four by four matrices are used (see Appendix I). In that case, set KC=1 and let KA be positive. The input consists of 8 cards with 4 values per card:

(1P4E14.6)

KC=1, KA>0

Q_{xx}	Q_{xy}	Q_{xz}	$Q_{x\varphi}$
Q_{yx}	Q_{yy}	Q_{yz}	$Q_{y\varphi}$
Q_{zx}	Q_{zy}	Q_{zz}	$Q_{z\varphi}$
$Q_{\varphi x}$	$Q_{\varphi y}$	$Q_{\varphi z}$	$Q_{\varphi \varphi}$
q_{xx}	q_{xy}	q_{xz}	$q_{x\varphi}$
q_{yx}	q_{yy}	q_{yz}	$q_{y\varphi}$
q_{zx}	q_{zy}	q_{zz}	$q_{z\varphi}$
$q_{\varphi x}$	$q_{\varphi y}$	$q_{\varphi z}$	$q_{\varphi \varphi}$

For the 4 pole homopolar generator it is found by the present analysis that $Q_{xz} = -Q_{yz} = Q_{\varphi x} = -Q_{\varphi y} = -q_{x\varphi} = -q_{y\varphi} = -q_{z\varphi} = -q_{\varphi \varphi}$ whereas the remaining gradients are zero.

For the heteropolar generator under load there are only timevarying forces and no moments, i.e. only Q_{xx} , Q_{yy} , q_x and q_y are different from zero. Furthermore, $Q_{xx} = Q_{yy}$ and $q_{xx} = q_{yy}$. In that case, set KC=0 and let KA be positive, and give 4 cards with two values per card:

(1P4E14.6)

KC=0, KA > 0

Q_{xx}	Q_{xy}
Q_{yx}	Q_{yy}
q_{xx}	q_{xy}
q_{yx}	q_{yy}

To increase the versatility of the program, there are three additional possibilities. If the timevarying magnetic forces are proportional to the slopes of the rotor and there are no moments, set KC=1 and let KA be positive, and give 4 cards with 2 values per card:

(1P4E14.6)

KC=1, KA > 0

$Q_{x\theta}$	$Q_{x\varphi}$
$Q_{y\theta}$	$Q_{y\varphi}$
$q_{x\theta}$	$q_{x\varphi}$
$q_{y\theta}$	$q_{y\varphi}$

If the timevarying magnetic moments are proportional to the rotor amplitudes and there are no forces, set KC=0 and let KA be negative, and give 4 cards with 2 values per card:

(1P4E14.6)

KC=0, KA < 0

$Q_{\theta x}$	$Q_{\theta y}$
$Q_{\varphi x}$	$Q_{\varphi y}$
$q_{\theta x}$	$q_{\theta y}$
$q_{\varphi x}$	$q_{\varphi y}$

Finally, if the timevarying magnetic moments are proportional to the rotor slopes and there are no forces, set KC=1 and let KA be negative, and give 4 cards with 2 values per card:

(1P4E14.6)

KC=1, KA < 0

$Q_{\theta\theta}$	$Q_{\theta\varphi}$
$Q_{\varphi\theta}$	$Q_{\varphi\varphi}$
$q_{\theta\theta}$	$q_{\theta\varphi}$
$q_{\varphi\theta}$	$q_{\varphi\varphi}$

Test Range of Magnetic Force Gradients

As explained under "Magnetic Force Data" in connection with eq. (L.1), the program searches for the threshold of instability as the zero points of two determinants by varying a parameter, Q_{ref} , which represents the gradients of the timevarying magnetic forces and moments. The actual gradients are equal to their respective input values times Q_{ref} . Thus, the input values for the gradients can be set equal to any value proportional to the actual gradients as long as the proportionality factor is the same for all the gradients, and when Q_{ref} equals this proportionality factor, the actual operating condition of the generator is encountered. In performing the stability calculation, Q_{ref} should in general be allowed a much wider range than the one corresponding to the actual generator operating condition. The range of Q_{ref} can be specified in two ways in the input. If $NQ=0$ (card 2, item 9) an initial value and a final value of Q_{ref} is specified together with an increment by which the program covers the specified range. Give one card with 3 values:

(1P5E14.6)

1. Initial value of Q_{ref}
2. Final value of Q_{ref}
3. Increment of Q_{ref}

If $NQ \geq 1$ (card 2, item 9), the stability determinants are evaluated at specified values of Q_{ref} . Give a total of NQ -values of Q_{ref} with 5 values per card according to the format (1P5E14.6).

Bearing Data, Fixed Geometry

When the bearings supporting the rotor are not of the tilting pad type, set $NPD=0$ (card 2, item 6). Then the dynamic reaction for each bearing is represented in terms of 8 coefficients. In other words, introduce a fixed x-y-coordinate system with origin in the steady-state position of the journal center, and let the corresponding journal amplitudes be x and y. Then the

bearing reactions F_x and F_y are:

$$F_x = -K_{xx}x - B_{xx}\frac{dx}{dt} - K_{xy}y - B_{xy}\frac{dy}{dt} \quad (L.2)$$

$$F_y = -K_{yx}x - B_{yx}\frac{dx}{dt} - K_{yy}y - B_{yy}\frac{dy}{dt}$$

where the K 's represent the bearing film stiffness and the B 's represent the damping. Values for typical bearings are given in Volume 3 and Volume 4.

If the bearing lubricant is incompressible (INC=0, card 2, item 7), there must be one card for each bearing (i.e. a total of NB cards, see card 2, Item 2). On each card are 8 values:

(8E9.2)

1. K_{xx} = spring coefficient, lbs/inch
2. ωB_{xx} = damping, lbs/inch
3. K_{xy} = Spring coefficient, lbs/inch
4. ωB_{xy} = Damping, lbs/inch
5. K_{yx} = spring coefficient, lbs/inch
6. ωB_{yx} = damping, lbs/inch
7. K_{yy} = spring coefficient, lbs/inch
8. ωB_{yy} = damping, lbs/inch.

Here, ω is the angular speed of the rotor in radians/sec., or, in other words, the four input values for damping gives the total damping at one per revolution, not the damping coefficients. This is in accordance with the way these coefficients are calculated from lubrication theory (see Volume 3).

If the bearing lubricant is a gas and, therefore, compressible, set INC=1 (card 2, item 7). Then the 8 bearing coefficients become functions of that frequency, ν , with which the rotor vibrates (the effect of squeeze number.) In calculating the stability determinants, the program calculates the rotors frequency response for as many frequencies as specified by the number of harmonics and, thus, it is necessary in the input to give the 8 bearing coefficients at

these frequencies. Let the timevarying magnetic forces have the frequency Ω radians/sec (the ratio $\frac{\Omega}{\omega}$ is given on the "Speed Data" card, item 4). Then it is necessary to evaluate the 8 bearing coefficients for $(NH+1)$ -frequencies (NH is given on card 2, item 8). These frequencies, ν , are:

$$\frac{\nu}{\omega} = 0, \frac{1}{2}\left(\frac{\Omega}{\omega}\right), \left(\frac{\Omega}{\omega}\right), \frac{3}{2}\left(\frac{\Omega}{\omega}\right), 2\left(\frac{\Omega}{\omega}\right), \dots, \frac{NH}{2}\left(\frac{\Omega}{\omega}\right) \quad (L.3)$$

where $\frac{\Omega}{\omega}$ is given by item 4 on the "Speed Data" card. It should be noted, that when ν is different from ω , the " ω " in the four damping values, ωB_{xx} , ωB_{xy} , ωB_{yx} and ωB_{yy} , is still the angular speed of the rotor.

Hence, for a compressible lubricant give $(NH+1)$ -cards per bearing where each card contains the values of the 8 bearing coefficients according to the same format as given above for an incompressible lubricant. The first card is for $\frac{\nu}{\omega} = 0$ and the last card for $\frac{\nu}{\omega} = \frac{NH}{2}\left(\frac{\Omega}{\omega}\right)$.

In this way, there will be a total of $NB \cdot (NH+1)$ cards with data for the bearing coefficients.

Bearing Data, Tilting Pad Bearing

When the bearings supporting the rotor are tilting pad bearings, set NPD equal to plus or minus the number of pads (card 2, item 6). The program assumes that the pads are arranged symmetrical with respect to a vertical axis so that the bearing operates with zero attitude angle. Hence, pads opposite each other operate under the same conditions and have the same dynamic coefficients, and it would be superfluous to repeat the same input for two pads. The program is set up to avoid such repetition of input. Furthermore, the tilting pad bearing may be oriented in two ways. Either the static bearing load passes between the two bottom pads or the load passes through the pivot of the bottom pad. In the first case, set NPD equal to the total number of pads. In the second case, set NPD equal to minus the total number of pads (i.e. the number of pads is equal to $|NPD|$). For each bearing the program requires input for NPDL number of pads:

If NPD is positive (load between pads): $\begin{cases} \text{NPD even, } NPD1=1/2 \text{ NPD} \\ \text{NPD odd, } NPD1=1/2 \cdot (NPD+1) \end{cases}$

If NPD is negative (load on pad): $\begin{cases} \text{NPD even, } NPD1=1/2 \cdot |NPD|+1 \\ \text{NPD odd, } NPD1=1/2 \cdot (|NPD|+1) \end{cases}$

Thus, for a four shoe bearing where the pivots are 45 degrees from the vertical load line, NPD=4 and NPD1=2. For a three shoe bearing where the vertical load line passes through the pivot of the bottom pad, NPD=-3 and NPD1=2.

Each pad film is represented by 8 dynamic coefficients as defined by eq. (L.2). However, here the x-axis passes through the pivot and the y-axis is perpendicular to the x-axis. The origin of the x-y-system changes from pad to pad.

For each bearing there must be data for NPD1 pads. The first card for a pad specifies the mass moment of inertia of the pad, the mass of the pad, the radial stiffness of the pivot support and the angle from the vertical load line to the pivot point:

(1P5E14.6)

1. The mass moment inertia of the pad with respect to the pitch axis divided by the square of the journal radius, lbs. The pitch axis is the axis parallel to the rotor axis through the pivot point.
2. The mass of the pad, lbs.
3. The radial stiffness of the pivot and its support, lbs/inch.
4. The angle from the static bearing load line to the pivot of the pad, degrees.

Then follows a card with the 8 pad film coefficients:

(8E9.2)

1. K_{xx} lbs/inch
2. ωB_{xx} lbs/inch
3. K_{xy} lbs/inch
4. ωB_{xy} lbs/inch

5. K_{yy} lbs/inch
6. ωB_{yy} lbs/inch
7. K_{yy} lbs/inch
8. ωB_{yy} lbs/inch

If the lubricant is incompressible (i.e. INC=0, card 2, item 7), there is only one card per pad with bearing coefficients. However, if the lubricant is compressible (INC=1), there must be (NH+1)-cards per pad with coefficients (for explanation, see "Bearing Data, Fixed Geometry"). Thus, for each pad there are either 2 input cards or (NH+2) input cards. Since the program requires data for NPD1 pads, there are either 2·NPD1 or (NH+2)·NPD1 cards per bearing. With NB bearings, the total bearing data input requires 2·NPD1·NB cards if INC=0, or (NH+2)·NPD1·NB cards if INC=1.

COMPUTER OUTPUT

Referring to the later given calculation where the output from the computer is shown, it is seen that the program output denotes the first couple of pages to a listing of the input values. Thereby any errors in the input are readily spotted. The input values are listed in the same sequence as the one in which they are given to the program. The only input data which are not repeated, is the card specifying the speed data and the card (or cards) specifying the test range of the magnetic force gradients.

After the listing of the input data follow the results of the calculations. For each rotor speed there will be a 3-column list. The first column lists the reference values of the magnetic force gradient (i.e. Q_{ref}). It is labeled "QXX" in the output. For each value of Q_{ref} , the values of the two stability determinants are given. The first determinant, labeled "EVEN DETERM.", is the stability determinant for even indices (see eq. (J.25), Appendix IX), and the second determinant, labeled "ODD DETERM." is the determinant for odd indices (see eq. (J.28), Appendix IX). Usually, the odd determinant is the one of greatest interest. It is the one that defines the instability zones centered at

$\frac{\Omega}{\omega} = 2, \frac{3}{2}, \frac{2}{3}, \frac{3}{4}, \dots$ (Ω is the frequency of the magnetic forces and $\omega_{critical}$ represents the critical speeds of the rotor-bearing system). If the speed of the rotor is ω , these instability zones are centered at:

$$\omega = \frac{\omega_{critical}}{(\frac{\Omega}{\omega})} \left\{ \begin{array}{l} 2 \\ \frac{3}{2} \\ \frac{2}{3} \\ \frac{3}{4} \\ \vdots \end{array} \right\}$$

where $(\frac{\Omega}{\omega})$ is the fixed ratio between the magnetic force frequency and the rotor speed, specified in the input (see Item 4, "Speed Data"). Of these instability zones, the first one is by far the most important except in very unusual circumstances.

The even determinant defines the instability zones centered at:

$$\omega = \frac{\omega_{critical}}{(\frac{\Omega}{\omega})} \left\{ \begin{array}{l} 1 \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{3} \\ \vdots \end{array} \right\}$$

Whenever one of the two determinants is zero, the corresponding value of Ω_{ref} defines a point on a boundary between a stable zone of operation and an unstable zone. The results do not indicate on which side of this boundary the system is stable or unstable. Hence it is necessary to perform calculations at sufficiently many rotor speeds to make it possible to draw up a stability map in the neighborhood of the operating speed.

SAMPLE CALCULATION

To illustrate the use of the stability computer program, a four pole homopolar generator with a turbine drive has been examined for stability. The rotor bearings are gas lubricated, and with a bearing stiffness of approximately 200,000 lbs/inch. The first three critical speeds are at 14,700 rpm, 16,000 rpm and 34,630 rpm. Because of the change in stiffness with frequency, these critical speeds are not the same for all harmonics. Calculations are performed over a speed range of

10,300 rpm to 20,300 rpm in increments of 1,000 rpm and the magnetic force gradient ranges from 0 to 1,000,000 in increments of 10,000. The resulting stability map is shown in Figure 20 . Three stability boundaries are well defined, labeled 1, 2 and 3, respectively. Boundaries 1 and 2 derive from zero-points of the odd determinant and are centered around the first and the second critical speed such that the rotor is unstable for operation between 15,200 rpm and 18,000 rpm. At the boundaries, the determinant actually crosses zero and the program then automatically interpolates to find the accurate value of Ω_{ref} at which the determinant becomes zero. Boundary No. 3 derives from zero points of the even determinant. In the output this determinant is never exactly zero but it is readily seen that the determinant has a minimum point whose value is equal to zero considering the numerical accuracy of the computation. In addition, discrete points of another boundary, labeled 4 in the map, have been obtained at 10,300 rpm, 11,300 rpm and 12,300 rpm. They derive from the zero-points of the odd determinant (the determinant has a minimum at these points). They are probably induced by excitation of 1/3 of the third critical speed in which case they would define an instability zone centered at 11,540 rpm, but more detailed calculations are needed to obtain a closer definition of this zone.

When the bearings are assigned their proper damping values it will be found that all the stability boundaries move upwards in the stability map such that, as an example, the two branches of boundary 1 meet and no longer reach the abscissa axis. However, if the rotors operating speed is within any of the indicated instability zones, although the bearing damping may stabilize the rotor, the stability margin must be considered small, and even if the rotor is not exactly unstable, the system is "weak" in the same sense as a system operating at its natural frequency whose amplitude is controlled solely by the damping available in the system. Therefore, operation within the instability zones indicated in Fig. 20 should be avoided.

INPUT FORM FOR COMPUTER PROGRAM
PN0351: THE STABILITY OF A ROTOR WITH TIMEVARYING MAGNETIC FORCES

Card 1 (72H)

Text

Card 2 (12I5)

1. NS = Number of rotor stations ($NS \leq 10$)
2. NB = Number of bearings ($NB \leq 10$)
3. KA $|KA|$ =Rotor station number at which magnetic forces act
 KA>0: forces only, no moments }
 KA<0: moments only, no forces } KC≥0
 KA>0, KC=-1: both forces and moments
4. KC KC=0: the magnetic forces or moments are proportional to amplitudes
 KC=1: the magnetic forces or moments are proportional to slope
 KC=-1: there are both magnetic forces and moments
5. NRP NRP=0: bearing pedestals are rigid, no pedestal input data
 NRP=1: flexible bearing pedestals, pedestal input data required
6. NPD NPD=0: fixed geometry bearings
 NPD≥1: number of pads in tilting pad bearing, load between pads
 NPD≤-1: $|NPD|$ =number of pads in tilting pad bearing, load on pad
7. INC INC=0: bearing lubricant is incompressible
 INC=1: bearing lubricant is compressible
8. NH = Number of frequency harmonics in stability calculation ($2 \leq NH \leq 20$)
9. NQ NQ=0: give range of Q_{ref} , program increments
 NQ≥1: give NQ-values of Q_{ref}
10. NSP = Number of speed ranges with accompanying data ($NSP \geq 1$)
11. NDIA NDIA=0: rotor impedance matrices not included in output
 NDIA=1: rotor impedance matrices included in output
 NDIA=-1: diagnostic
12. INP INP=0: more input follows, starting from card 1
 INP=1: last set of input data

Card 3 (1P5E14.6)

1. YM = Youngs modulus for shaft material, lbs/in^2
2. DMST = Weight density of shaft material, lbs/in^3
3. SHM = αG , where G is shear modulus, lbs/in^2 , and α is shape factor for shear.

Rotor Data (8E9.2)

Give NS cards with 7 numbers on each card:

1. Mass at rotor station, lbs.
2. Polar mass moment of inertia at rotor station, lbs-in^2
3. Transverse mass moment of inertia at rotor station, lbs-in^2
4. Length of shaft section to next station, inch
5. Outer shaft diameter for cross-sectional moment of inertia, inch
6. Outer shaft diameter for shaft mass, inch
7. Inner shaft diameter, inch.

Bearing Stations (12I5)

List the rotor stations at which there are bearings, in total NB stations

Pedestal Data (8E9.2)

This data only applies when NRP=1 (card 2, item 5). Give a total of NB cards with 6 values per card:

1. Pedestal mass, x-direction, lbs.
2. Pedestal stiffness, x-direction, lbs/inch
3. Pedestal damping, x-direction, lbs-sec/inch
4. Pedestal mass, y-direction, lbs.
5. Pedestal stiffness, y-direction, lbs/inch
6. Pedestal damping, y-direction, lbs-sec/inch

Note: The following data must be repeated NSP-times (Card 2, Item 10)

Speed Data (1P5E14.6)

Give one card with 5 values:

1. Initial speed, rpm
2. Final speed, rpm
3. Speed increment, rpm
4. Ratio of magnetic force frequency to rotor speed
5. Scale factor for determinant (set equal to mass of rotor)

Magnetic Force Data

Card (1P5E14.6)

1. Static gradient of magnetic force, Q_0 , lbs/in
2. Static gradient of magnetic moment, Q'_0 , lbs-inch/radian

Cards (1P4E14.6)

a. If KC=1 (card 2, item 4), give 8 cards with 4 values per card:

Q_{xx}	Q_{xy}	Q_{xz}	Q_{xp}
Q_{yx}	Q_{yy}	Q_{yz}	Q_{yp}
Q_{zx}	Q_{zy}	Q_{zz}	Q_{zp}
Q_{px}	Q_{py}	Q_{pz}	Q_{pp}
q_{xx}	q_{xy}	q_{xz}	q_{xp}
q_{yx}	q_{yy}	q_{yz}	q_{yp}
q_{zx}	q_{zy}	q_{zz}	q_{zp}
q_{px}	q_{py}	q_{pz}	q_{pp}

These are the gradients of the timevarying magnetic forces and moments:

$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}, q_{xx}, q_{xy}, q_{yx}, q_{yy}$ in lbs/inch
 $Q_{xz}, Q_{xp}, Q_{yz}, Q_{yp}, q_{xz}, q_{xp}, q_{yz}, q_{yp}$ in lbs/radian
 $Q_{zx}, Q_{zy}, Q_{py}, Q_{pp}, q_{zx}, q_{zy}, q_{py}, q_{pp}$ in lbs-inch/inch
 $Q_{zz}, Q_{zp}, Q_{pq}, Q_{pp}, q_{zz}, q_{zp}, q_{pq}, q_{pp}$ in lbs-inch/radian

b. If KC=0, give 4 cards with 2 values per card

<u>KA>0</u>	<u>KA<0</u>
Q_{xx} Q_{xy}	Q_{ox} Q_{oy}
Q_{yx} Q_{yy}	Q_{px} Q_{py}
q_{xx} q_{xy}	q_{ox} q_{oy}
q_{yx} q_{yy}	q_{px} q_{py}

c. If KC=1, give 4 cards with 2 values per card

<u>KA>0</u>	<u>KA<0</u>
Q_{xz} Q_{xp}	Q_{oe} Q_{op}
Q_{yz} Q_{yp}	Q_{pe} Q_{pp}
q_{xz} q_{xp}	q_{oe} q_{op}
q_{yz} q_{yp}	q_{pe} q_{pp}

Test Range of Magnetic Force Gradients (1P5E14.6)

a. If NQ=0 (card 2, item 9): Give 1 card with 3 values:

1. Initial value of Q_{ref}
2. Final value of Q_{ref}
3. Increment of Q_{ref}

b. If $NQ \neq 1$: Give cards with 5 values of Q_{ref} per card, total NQ-values

Bearing Data, Fixed Geometry (8E9.2)

Applies when NPD=0 (card 2, item 6). If the lubricant is incompressible (INC=0; card 2, item 7), give one card per bearing. If the lubricant is compressible (INC=1), give $(NH+1)$ -cards per bearing (NH is item 8, card2). Each card gives a set of 8 bearing coefficients:

1. Spring coefficient K_{xx} , lbs/inch
2. Damping ωB_{xx} , lbs/inch
3. Spring coefficient K_{xy} , lbs/inch
4. Damping ωB_{xy} , lbs/inch

5. Spring coefficient K_{yx} , lbs/inch
6. Damping ωB_{yx} , lbs/inch
7. Spring coefficient K_{yy} , lbs/inch
8. Damping ωB_{yy} , lbs/inch

Bearing Data, Tilting Pad Bearing

Applies when NPD $\neq 0$ (card 2, item 6). Define the number NPD1 by:

if $NPD \geq 1$ (load between pads): $\begin{cases} NPD \text{ even, then: } NPD1=1/2 \cdot NPD \\ NPD \text{ odd, then: } NPD1=1/2 \cdot (NPD+1) \end{cases}$

if $NPD \leq -1$ (load on pad): $\begin{cases} |NPD| \text{ even, then: } NPD1=1/2 \cdot |NPD| + 1 \\ |NPD| \text{ odd, then: } NPD1=1/2 \cdot (|NPD| + 1) \end{cases}$

NPD1 is the number of pads for which input is required per bearing. If the lubricant is incompressible (INC=0; card 2, item 7), give two cards per pad. If the lubricant is compressible (INC=1), give $(NH+2)$ -cards per pad. In either case the first card is:

(1P5E14.6)

1. Pitch mass moment of inertia divided by the square of the journal radius, lbs
2. Pad mass, lbs
3. Radial stiffness of pivot support, lbs/inch
4. Angle from bearing load line to pivot point, degrees.

Then follow 1 card if INC=0, or $(NH+1)$ -cards if INC=1, with the 8 dynamic coefficients for the pad:

Cards (8E9.2)

1. Spring coefficient K_{xx} , lbs/inch
2. Damping ωB_{xx} , lbs/inch
3. Spring coefficient K_{xy} , lbs/inch
4. Damping ωB_{xy} , lbs/inch

5. Spring coefficient K_{yx} , lbs/inch
6. Damping ωB_{yx} , lbs/inch
7. Spring coefficient K_{yy} , lbs/inch
8. Damping ωB_{yy} , lbs/inch

These (NH+2)-cards must be repeated NPD1 times per bearing, and there must be one complete set for each bearing (there are NB bearings).

07/03/67

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C 3-14-67 J.LUND MECHANICAL TECHNOLOGY INC.
C PN351=ROTOR STABILITY WITH MAGNETIC FORCES
C DIMENSION RM(100),RIP(100),RL(100),RS(100),RW(100),RD(100),
C DVXA(100),DVXB(100),DVXC(100),DVXD(100),DVYA(100),DVYB(100),
C DVYC(100),DVYC(100),DVUX(100),DVUY(100),DMUX(100),DMUY(100),
C B31(100),B2(100),B3(100),B4(100),B5(100),B6(100),B7(100),B8(100),
C 4B9(100),B10(100),PMX(10),PKX(1C),PDX(10),PMY(10),PKY(10),PDY(10),
C S5XX(10),DXX(10),SXY(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),
C 6QLST(200),LB(10),PMXA(100),RIT(100)
C DIMENSION BKXX(10,21),RCXX(10,21),BKXY(10,21),BCXY(10,21),
C 1BKYX(10,21),BCYX(10,21),BKYY(10,21),BCYY(10,21),PMIN(10,5),
C 2PADM(10,5),PACK(10,5),PANG(10,5),DEVN(8,8),DEDD(8,8),CMR(4,8),
C 3CME(4,8),AMR(8,8),AME(8,8),MR(8,8),WE(8,8),WA(8,8),WB(4,4),
C 4WCC(8,1),UR(8,8),UE(8,8),EMR(4,4),EME(4,4),WC(4,4),WSQ(4,4)
C DIMENSION PKXX(10,5,21),PCXX(10,5,21),PKXY(10,5,21),
C 1PCXY(10,5,21),PKYX(10,5,21),PCYX(10,5,21),PKYY(10,5,21),
C 2PCYY(10,5,21),GR(4,4,21),GE(4,4,21)
C WRITE(6,99)
190 READ(5,100)
READ(5,101) NS,NB,KA,KC,NRP,NPD,INC,NH,NQ,NSP,NDIA,IMP      2
READ(5,102) YM,CNST,SHM                                     3
WRITE(6,100)
WRITE(6,103)
WRITE(6,104)NS,NB,KA,KC,NRP,NPD,INC,NH,NQ,NSP,NDIA,IMP      16
WRITE(6,105)
WRITE(6,102)YM,DNST,SHM                                     17
DNST=CNST/386.069                                         18
NS1=NS-1
NH1=NH+1
IF(KC) 196,195,195
195 KQ1=4
KQ2=2
KQ3=6
GO TO 197
196 KQ1=8
KQ2=4
KQ3=8
197 IF(KA) 198,199,199
198 KB=-KA
GO TO 200
199 KB=KA
200 WRITE(6,110)
WRITE(6,108)
DO 203 J=1,NS
READ(5,106) RM(J), RIP(J), RIT(J), RL(J), RS(J), RW(J), RD(J)   37
WRITE(6,107)J,RM(J),RIP(J),RIT(J),RL(J),RS(J),RW(J),RD(J)       38
RM(J)=RM(J)/386.069
RIP(J)=RIP(J)/386.069
RIT(J)=RIT(J)/386.069
C1=0.049087385*YM*(RS(J)**4-RD(J)**4)                         41
RW(J)=0.78539816*DNST*(RW(J)**2-RD(J)**2)                      49
C2=1.5707963*SHM*(RS(J)**2-RD(J)**2)
RS(J)=C1
IF(C2) 202,202,201
201 RD(J)=C1/C2

```

DO ONE

- EPM SOURCE STATEMENT - EPM(3) -

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GO TO 203
202 RD(J)=0.0
203 CONTINUE
READ (5,101) LB(J),J=1,NB
WRITE(6,109)
WRITE(6,101)(LB(J),J=1,NB)
IF(NRP) 210,210,204
204 WRITE(6,111)
WRITE(6,112)
DO 205 J=1,NB
READ (5,106) PMX(J), PKX(J), POX(J), PMY(J), PKY(J), POY(J)
WRITE(6,107) LB(J),PMX(J),PKX(J),POX(J),PMY(J),PKY(J),POY(J)
PMX(J)=PMX(J)/386.069
205 PMY(J)=PMY(J)/386.069
210 IF(NPD) 214,217,211
211 IF((NPD/2)*2-NPD) 213,212,212
212 NPD1=NPD/2
NPD2=-2
GO TO 217
213 NPD1=(NPD+1)/2
NPD2=-1
GO TO 217
214 NPD1=-NPD
IF((NPD1/2)*2-NPD1) 216,215,215
215 NPD1=NPD1/2+1
NPD2=0
GO TO 217
216 NPD1=(NPD1+1)/2
NPD2=1
217 MSP1=1
230 READ (5,102) SPST, SPFN, SPIN, SFR, SCF
READ (5,102) QZ, QZP
WRITE(6,113)
WRITE(6,114)
WRITE(6,102) QZ, QZP
WRITE(6,115)
DO 218 I=1,KQ2
READ (5,133) (WQ(I,J),J=1,KQ2)
WRITE(6,133)(WQ(I,J),J=1,KQ2)
218 CONTINUE
WRITE(6,126)
DO 219 I=1,KQ2
READ (5,133) (WSQ(I,J),J=1,KQ2)
WRITE(6,133)(WSQ(I,J),J=1,KQ2)
219 CONTINUE
IF(NQ) 231,231,232
231 READ (5,102) QST, QFN, QINC
GO TO 233
232 READ (5,102) (QLST(J),J=1,NQ)
233 SFR1=0.052359878*SFR
WRITE(6,116)
IF(INC) 242,241,242
241 K1=1
GO TO 243
242 K1=NH1
243 DO 255 J=1,NB

CVE - EFM SOURCE STATEMENT, > IFN(SI) -

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244 WRITE(6,117)LR(J)
IF(NPC) 257,244,250
244 WRITE(6,118)
DO 245 I=1,K1
K2=I-1
READ (5,106) BKXX(J,I), BCXX(J,I), BKXY(J,I), BCXY(J,I),
BKYY(J,I), BCYX(J,I), BKYY(J,I), BCYY(J,I)
WRITE(6,107)K2,BKXX(J,I),BCXX(J,I),BKXY(J,I),BCXY(J,I),BKYY(J,I),
BCYX(J,I),BKYY(J,I),BCYY(J,I)
245 CONTINUE
IF(INC) 255,246,255
246 DO 247 I=2,NH1
BKXX(J,I)=BKXX(J,1)
BCXX(J,I)=BCXX(J,1)
BKXY(J,I)=BKXY(J,1)
BCXY(J,I)=BCXY(J,1)
BKYY(J,I)=BKYY(J,1)
BCYX(J,I)=BCYX(J,1)
BKYY(J,I)=BKYY(J,1)
247 BCYY(J,I)=BCYY(J,1)
GO TO 255
250 DO 254 K=1,NPC1
WRITE(6,119)K
WRITE(6,120)
READ (5,102) PMIN(J,K), PADM(J,K), PADK(J,K), PANG(J,K)
WRITE(6,102)PMIN(J,K),PADM(J,K),PADK(J,K),PANG(J,K)
PMIN(J,K)=PMIN(J,K)/386.069
PADM(J,K)=PADM(J,K)/386.069
PANG(J,K)=0.317453233*PANG(J,K)
WRITE(6,118)
DO 251 I=1,K1
K2=I-1
READ (5,106) PKXX(J,K,I), PCXX(J,K,I), PKXY(J,K,I), PCXY(J,K,I),
PKYY(J,K,I), PCYX(J,K,I), PKYY(J,K,I), PCYY(J,K,I)
WRITE(6,107)K2,PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),
PKYY(J,K,I),PCYX(J,K,I),PKYY(J,K,I),PCYY(J,K,I)
251 CONTINUE
IF(INC) 254,252,254
252 DO 253 I=2,NH1
PKXX(J,K,I)=PKXX(J,K,1)
PCXX(J,K,I)=PCXX(J,K,1)
PKXY(J,K,I)=PKXY(J,K,1)
PCXY(J,K,I)=PCXY(J,K,1)
PKYY(J,K,I)=PKYY(J,K,1)
PCYX(J,K,I)=PCYX(J,K,1)
PKYY(J,K,I)=PKYY(J,K,1)
253 PCYY(J,K,I)=PCYY(J,K,1)
254 CONTINUE
255 CONTINUE
SPD=SPST
260 WRITE(6,121)SPD
SCF1=0.10471976*SPD
SCF1=SCF1*SCF1*SCF1
DO 519 IH=1,NH1
NF=IH-1
HN=NF

ONE

- EPM SOURCE STATEMENT - IFR(SI) -

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```
FQ0=HN*SPD*SFR1
FQ2=FQ0+FQ0
HN1=HN/2.0*SFR
IF(NDIA) 401,402,401
401 WRITE(6,122)NF,FQ0
      WRITE(6,123)
      WRITE(6,124)
402 DO 425 J=1,NB
      IF(NPD)404,403,404
403 D1=BKXX(J,IH)
      D2=BCXX(J,IH)*HN1
      D3=BKXY(J,IH)
      D4=BCXY(J,IH)*HN1
      D5=BKYY(J,IH)
      D6=BCYY(J,IH)*HN1
      D7=BKYY(J,IH)
      D8=BCYY(J,IH)*HN1
      GO TO 416
404 D1=0.0
      D2=0.0
      D3=0.0
      D4=0.0
      D5=0.0
      D6=0.0
      D7=0.0
      D8=0.0
      DO 415 I=1,NPC1
      C1=FQ2*PMIN(J,I)
      A1=PKXX(J,I,IH)
      A2=PCXX(J,I,IH)*HN1
      A3=PKXY(J,I,IH)
      A4=PCXY(J,I,IH)*HN1
      A5=PRYX(J,I,IH)
      A6=PCYX(J,I,IH)*HN1
      A7=PKYY(J,I,IH)
      A8=PCYY(J,I,IH)*HN1
      C2=A7-C1
      C3=PADK(J,I)-FQ2*PADM(J,I)
      C4=A1+C3
      C5=C4*C2-A2*A8-A3*A5+A4*A6
      C6=C4*A8+C2*A2-A3*A6-A5*A4
      C7=C5+C5+C6+C6
      C11R=C3*(C5*C2+C6*A8)/C7
      C11E=C3*(C5*A8-C6*C2)/C7
      C12R=C1*(C5*A3+C6*A4)/C7
      C12E=C1*(C5*A4-C6*A3)/C7
      C21R=-C3*(C5*A5+C6*A6)/C7
      C21E=C3*(C6*A5-C5*A6)/C7
      C22R=-C1*(C5*C4+C6*A2)/C7
      C22E=C1*(C6*C4-C5*A2)/C7
      DKXX=C11R*A1-C11E*A2+C21R*A3-C21E*A4
      DCXX=C11R*A2+C11E*A1+C21R*A4+C21E*A3
      DKXY=C12R*A1-C12E*A2+C22R*A3-C22E*A4
      DCXY=C12R*A2+C12E*A1+C22R*A4+C22E*A3
      DKYX=C11R*A5-C11E*A6+C21R*A7-C21E*A8
      DCYK=C11R*A6+C11E*A5+C21R*A8+C21E*A7
```

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CSE - EBN SOURCE STATEMENT, F. IFNISI -

07/03/67

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DKYY=C12E*A5+C12E*A6+C22E*A7-C22E*A8
DCYY=C12E*A6+C12E*A5+C22E*A8+C22E*A7
C3=PAHG(J,1)
C1=CUS(C3)
C2=SIN(C3)
C4=C1+C1
C5=C2+C2
A1=DKXX*C4+DKYY*C5
A2=DCXX*C4+DCYY*C5
A3=DKXY*C4-DKYY*C5
A4=DCXY*C4-DCYX*C5
A5=DKYY*C4-DKXY*C5
A6=DCYY*C4-DCXY*C5
A7=DKYY*C4+DKXX*C5
A8=DCYY*C4+DCXX*C5
IF(NPC2+1) 410,405,405
405 IF(I-1) 406,406,407
406 IF(NPD2) 409,409,410
407 IF(I-NPD1) 410,408,408
408 IF(NPC2) 410,409,409
409 D1=D1+A1
D2=D2+A2
D3=D3+A3
D4=D4+A4
D5=D5+A5
D6=D6+A6
D7=D7+A7
D8=D8+A8
GO TO 415
410 D1=D1+A1+A1
D2=D2+A2+A2
D3=D3+A3+A3
D4=D4+A4+A4
D5=D5+A5+A5
D6=D6+A6+A6
D7=D7+A7+A7
D8=D8+A8+A8
415 CONTINUE
416 IF(INDIA) 417,420
417 WRITE(5,107)LB/ 2,03,04,05,06,07,08
420 IF(NRP) 421,+21
421 SX(X(J)=D1:
DX(X(J)=D2
SX(Y(J)=D3
DX(Y(J)=D4
SY(X(J)=D5
DY(X(J)=D6
SY(Y(J)=D7
DY(Y(J)=D8
GO TO 425
422 C1=PKX(J)-FQ2*PMX(J)
C2=PKY(J)-FQ2*PMY(J)
C3=FRQ*PDX(J)
C4=FRQ*PDY(J)
C5=D1+C1
C6=D7+C2

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ONE

- EPN SOURCE STATEMENT - IPN151 -

```
C7=02*C3
C8=08*C4
A1=C5*C6-C7*C8-D3*D5+D4*D6
A2=C3*C8+C6*C7-D3*D6-D4*D5
C9=A1*A1+A2*A2
A3=C1*C6-C3*C8
A4=C1*C8+C3*C6
C11R=(A3*A1+A4*A2)/C9
C11E=(A4*A1-A3*A2)/C9
A3=C2*D3-C4*D4
A4=C2*D4+C4*D3
C12R=-(A3*A1+A4*A2)/C9
C12E=-(A4*A1-A3*A2)/C9
A3=C1*D5-C3*D6
A4=C1*D6+C3*D5
C21R=-(A3*A1+A4*A2)/C9
C21E=-(A4*A1-A3*A2)/C9
A3=C2*C5-C4*C7
A4=C2*C7+C4*C5
C22R=(A3*A1+A4*A2)/C9
C22E=(A4*A1-A3*A2)/C9
SXX(J)=C11R*D1-C11E*D2+C21R*D3-C21E*D4
DXX(J)=C11R*D2+C11E*D1+C21R*D4+C21E*D3
SXY(J)=C12R*D1-C12E*D2+C22R*D3-C22E*D4
DXY(J)=C12R*D2+C12E*D1+C22R*D4+C22E*D3
SYX(J)=C11R*D5-C11E*D6+C21R*D7-C21E*D8
DYX(J)=C11R*D6+C11E*D5+C21R*D8+C21E*D7
SYV(J)=C12R*D5-C12E*D6+C22R*D7-C22E*D8
DYY(J)=C12R*D6+C12E*D5+C22R*D8+C22E*D7
```

425 CONTINUE

DO 449 J=1,NS1

C1=RS(J)

C2=FQ2*RM(J)

C3=RD(J)

C4=C2/C1

C5=SQRT(C4)

C6=SQRT(C5)

C7=RL(J)

IF(C6*C7-0.03) 441,441,442

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413

441 C8=C2*C7

B1(J)=1.0

B2(J)=1.0

B3(J)=C7

B6(J)=C7/C1

B4(J)=B6(J)/2.0*C7

B7(J)=B4(J)/3.0*C7-C3*C7/C1*2.0

B5(J)=C2*B7(J)

B8(J)=C8

B9(J)=C8/2.0*C7

B10(J)=B9(J)/3.0*C7

GO TO 449

442 C8=C3*C3*C4

C9=C3*C5

IF(C8-0.0002) 443,443,444

443 C8=1.0+0.5*C8

GO TO 445

CYE - EF4 SOURCE STATEMENT - IFHISI -

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444 CB=SQRT(1.0+C8)
445 A5=C5*(C8-C9)
A6=C5*(C9+C9)
A9=A5+A6
A3=SQRT(A5)
A4=SQRT(A6)
A7=A3*A5
A8=A4*A6
A1=A3*C7
A2=A4*C7
D1=COS(A2)/A9
D2=SIN(A2)/A9
D5=EXP(A1)
D4=1.0/D5
D3=0.5*(D5+D4)/A9
D4=0.5*(D5-D4)/A9
B1(J)=A5*D3+A6*D1
B2(J)=A6*D3+A5*D1
B3(J)=A3*D4+A4*D2
B8(J)=C2*B3(J)
B4(J)=(D3-D1)/C1
B9(J)=C2*(D3-C1)
B5(J)=C5*(A4*C4-A3*C2)
B10(J)=C1*B5(J)
C8=C1*C5
B6(J)=(A9*D4+A7*C2)/C8
B7(J)=(A7*D4-A8*C2)/C2
449 CONTINUE
DO 455 J=1,NS
C1=FQ2*RIM(J)
DVXA(J)=C1
DVYA(J)=C1
DVXB(J)=0.0
DVXC(J)=0.0
DVXD(J)=0.0
DVYB(J)=0.0
DVYC(J)=0.0
DVYD(J)=0.0
DMXA(J)=FQ2*RIT(J)
DVUX(J)=0.0
DVUY(J)=0.0
DMUX(J)=0.0
455 DMUY(J)=0.0
DO 456 J=1,NB
K1=L8(J)
DVXA(K1)=DVXA(K1)-SXX(J)
DVXB(K1)=DXX(J)
DVXC(K1)=SXY(J)
DVXD(K1)=DXY(J)
DVYA(X1)=DVYA(K1)-SYY(J)
DVYB(K1)=DYY(J)
DVYC(K1)=SYX(J)
456 DVYD(K1)=DXY(J)
DVXA(K8)=DVXA(K8)+QZ
DVYA(K8)=DVYA(K8)+QZ
DMXA(K8)=DMXA(K8)+QZP

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ONE - EPW SOURCE STATEMENT - IPH(S) -

07/09/67

DO 485 I=L,KQ3
BXJC=0.0
BMAX=0.0
BNYC=0.0
BNYS=0.0
VXC=0.0
VXS=0.0
VYC=0.0
VYS=0.0
XC=0.0
KS=0.0
YC=0.0
YS=0.0
DXC=0.0
DXS=0.0
DYC=0.0
DYS=0.0
DVUX(K8)=0.0
DVUY(K8)=0.0
DMUX(K8)=0.0
DMUY(K8)=0.0
GO TO(461,462,463,464,465,469,468,472),1
461 XC=0.001
GO TO 475
462 YC=0.001
GO TO 475
463 DXC=0.001
GO TO 475
464 DYC=0.001
GO TO 475
465 IF(KC) 467,466,466
466 IF(KA) 468,467,467
467 DVUX(K8)=1.0
GO TO 475
468 DMUX(K8)=1.0
EJ TO 475
469 IF(KC) 471,470,470
470 IF(XA) 472,471,471
471 DVUY(K8)=1.0
GO TO 475
472 DMUY(K8)=1.0
473 DO 483 J=1,NS
C1=DMXA(J)
C2=FRO+SPD+RIP(J)
IF(J-K8) 477,476,477
476 CMR(1,I)=XC
CME(1,I)=XS
CMR(2,I)=YC
CME(2,I)=YS
CMR(3,I)=DXC
CME(3,I)=DXS
CMR(4,I)=DYC
CME(4,I)=DYS
477 A1=BMXC-C1*DXC-C2*DYS-DMUX(J)
A2=BMAX-C1*DXS+C2*DYC
A3=BNYC-C1*DYC+C2*DXS-DMUY(J)

ONE - EFY SOURCE STATEMENT - IEN(S) -

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A6=8MYS-C1*DYS-C2*DYC
A5=VXC+DVXA(J)*XC+DXB(J)*XS-DVC(J)*YC+DVB(J)*YS+DVUX(J)
A6=VAS-DVB(J)*XC+DVXA(J)*XS-DVA(J)*YC-DVC(J)*YS
A7=VTC-DYYC(J)*XC+DVYD(J)*XS+DVYA(J)*YC+DYYD(J)*YS+DVUY(J)
A8=VVS-DYYD(J)*XC-DVYC(J)*XS-DVYB(J)*YC+DVYA(J)*YS
IF(INS-J) 46J,452,473
478 C1=XC
C2=XS
C3=YC
C4=YS
BMXC=C1*B9(J)+DXC*B10(J)+A1*B2(J)+A5*B3(J)
BMXS=C2*B9(J)+DXS*B10(J)+A2*B2(J)+A6*B3(J)
BMYC=C3*B9(J)+DYC*B10(J)+A3*B2(J)+A7*B3(J)
BMYS=C4*B9(J)+DYS*B10(J)+A4*B2(J)+A8*B3(J)
VXC=C1*B8(J)+CXC*B9(J)+A1*B5(J)+A5*B1(J)
VXS=C2*B8(J)+CXS*B9(J)+A2*B5(J)+A6*B1(J)
VYC=C3*B8(J)+CYC*B9(J)+A3*B5(J)+A7*B1(J)
VYS=C4*B8(J)+CYS*B9(J)+A4*B5(J)+A8*B1(J)
XC=C1*B1(J)+DXC*B3(J)+A1*B4(J)+A5*B7(J)
XS=C2*B1(J)+DXS*B3(J)+A2*B4(J)+A6*B7(J)
YC=C3*B1(J)+DYC*B3(J)+A3*B4(J)+A7*B7(J)
YS=C4*B1(J)+DYS*B3(J)+A4*B4(J)+A8*B7(J)
DXC=C1*B5(J)+CXC*B2(J)+A1*B6(J)+A5*B4(J)
DXS=C2*B5(J)+CXS*B2(J)+A2*B6(J)+A6*B4(J)
DYC=C3*B5(J)+CYC*B2(J)+A3*B6(J)+A7*B4(J)
DYS=C4*B5(J)+CYS*B2(J)+A4*B6(J)+A8*B4(J)
480 CONTINUE
AMR(I,I)=A1
AME(I,I)=A2
AMR(2,I)=A3
AME(2,I)=A4
AMR(3,I)=A5
AME(3,I)=A6
AMR(4,I)=A7
AME(4,I)=A8
485 CONTINUE
DO 486 J=1,4
DO 486 I=1,4
WR(I,J)=AMR(I,J)
486 WE(I,J)=AME(I,J)
CALL MATINV(WR,4,WCC,0,ENV,1D)          666
GO TO(481,460),IC
460 WRITE(6,130)                         668
WRITE(6,131)NF,FQ
GO TO 519
481 IF(NF) 457,482
457 DO 459 I=1,4
DO 458 J=1,4
AME(I,J)=0.0
UR(I,J)=WR(I,J)
458 UE(I,J)=0.0
GO TO 459
482 CALL MATINV(WE,4,WCC,0,DED,1D)        687
GO TO(483,457),IC
483 DO 488 I=1,4
DO 488 J=1,4

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ONE - EFW SOURCE STATEMENT - (FNIS) -

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C1=0.0
DO 487 K=1,4
487 C1=C1+WR(I,K)*ANE(I,J)+WE(I,K)*AMR(K,J)
488 MA(I,J)=C1
CALL MATINV(MA,4,MCC,0,OVN,1D)
GO TO 1702,7011,10
701 WRITE(6,137)
WRITE(6,131)NFM,FRQ
GO TO 519
702 DO 490 I=1,4
DO 490 J=1,4
C1=0.0
C2=0.0
DO 489 K=1,4
C1=C1+WA(I,K)*WE(K,J)
489 C2=C2-WA(I,K)*WR(K,J)
UR(I,J)=C1
490 UE(I,J)=C2
499 DO 492 I=1,4
DO 492 J=1,4
C1=0.0
C2=0.0
DO 491 K=1,4
C1=C1+CMR(I,K)*UR(K,J)-CME(I,J)*UE(K,J)
491 C2=C2+CMR(I,K)*UE(K,J)+CME(I,K)*UR(K,J)
WR(I,J)=C1
492 WE(I,J)=C2
DO 494 I=1,4
DO 494 J=1,KQ2
C1=CMR(I,J+4)
C2=CME(I,J+4)
DO 493 K=1,4
C1=C1-WR(I,K)*AMR(K,J+4)+WE(I,K)*AME(K,J+4)
493 C2=C2-WR(I,K)*AME(K,J+4)-WE(I,K)*AMR(K,J+4)
UR(I,J)=C1
494 UE(I,J)=C2
IF(KC) 500,499,499
499 DO 497 I=1,1
DO 497 J=1,2
IF(KC) 500,495,496
495 EMR(I,J)=UR(I,J)
EME(I,J)=UE(I,J)
GO TO 497
496 EMR(I,J)=UR(I+2,J)
EME(I,J)=UE(I+2,J)
497 CONTINUE
C1=EMR(1,1)*EMR(2,2)-EMR(1,2)*EMR(2,1)-EME(1,1)*EME(2,2)-
1EME(1,2)*EME(2,1)
C2=EMR(1,1)*EME(2,2)+EMR(2,2)*EME(1,1)-EMR(1,2)*EME(2,1)-
1EMR(2,1)*EME(1,2)
C3=(C1+C2*C2)*SCF1
C1=C1/C3
C2=C2/C3
GR(1,1,IH)=C1*EMR(2,2)+C2*EME(2,2)
GR(1,2,IH)=-C1*EMR(1,2)-C2*EME(1,2)
GR(2,1,IH)=-C1*EMR(2,1)-C2*EME(2,1)

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GR(2,2,IH)= C1*EPR(1,1)+C2*EME(1,1)
GE(1,1,IH)= C1*EPE(2,2)-C2*EMR(2,2)
GE(1,2,IH)=-C1*EMF(1,2)+C2*EMR(1,2)
GE(2,1,IH)=-C1*EMF(2,1)+C2*EMR(2,1)
GE(2,2,IH)= C1*EPE(1,1)-C2*EMR(1,1)
GO TO 518
500 DO 501 I=1,4
DO 501 J=1,4
WR(I,J)=UR(I,J)
501 WE(I,J)=UE(I,J)
.CALL MATINV(WR,4,WCC,0,DVN,1D) 816
GO TO (503,502),ID
502 WRITE(6,132)
WRITE(6,131)NF,FRQ
GO TO 519
503 IF(NF) 504,504,506
504 DO 505 I=1,4
DO 505 J=1,4
GE(I,J,IH)=0.0
505 GR(I,J,IH)=WR(I,J)/SCF1
GO TO 518
506 CALL MATINV(WE,4,WCC,0,DED,1D) 836
GO TO (507,504),ID
507 DO 509 I=1,4
DO 509 J=1,4
C1=0.0
DO 508 K=1,4
508 C1=C1+WR(I,K)*UE(K,J)+WE(I,K)*UR(K,J)
509 WA(I,J)=C1
.CALL MATINV(WA,4,WCC,0,DVN,1D) 855
GO TO (704,703),ID
703 WRITE(6,138)
WRITE(6,131)NF,FRQ
GO TO 519
704 DO 517 I=1,4
DO 517 J=1,4
C1=0.0
C2=0.0
DO 516 K=1,4
C1=C1+WA(I,K)*WE(K,J)
516 C2=C2-WA(I,K)*WR(K,J)
GR(I,J,IH)=C1/SCF1
517 GE(I,J,IH)=C2/SCF1
518 IF(INDIA) 498,519,498
498 WRITE(6,125)
WRITE(6,127)
WRITE(6,133)((GR(I,J,IH),J=1,4),I=1,4) 880
WRITE(6,128)
WRITE(6,133)((GE(I,J,IH),J=1,4),I=1,4) 881
882
890
891
519 CONTINUE
K3=0
K4=0
K5=0
K6=0
K7=1
WRITE(6,129)NCIA 906

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QX1=QST-QINC
NQ1=1
510 IF(NQ) 511,511,512
511 QX1=QX1+QINC
      IF(QFM+0.000001-QX1) 599,599,515
512 IF(NQ-NQ1) 599,513,513
513 QX1=QLST(NQ1)
      NQ1=NQ1+1
515 QX2=0.5*QX1/SCF1
      DO 521 I=1,KQ2
      DO 521 J=1,KQ2
      WA(I,J)=QX2*WQ(I,J)
      WB(I,J)=-QX2*WSQ(I,J)
      AMR(I,J)=0.0
      AME(I,J)=0.0
      CMR(I,J)=0.0
521 CME(I,J)=0.0
      DO 553 IH=2,NH1
      K1=NH1+2-IH
      IF((K1/2)*2-K1) 523,525,525
523 K2=0
      DO 524 I=1,KQ2
      DO 524 J=1,KQ2
      UR(I,J)=AMR(I,J)
524 UE(I,J)=AME(I,J)
      GO TO 527
525 K2=1
      DO 526 I=1,KQ2
      DO 526 J=1,KQ2
      UR(I,J)=CMR(I,J)
526 UE(I,J)=CME(I,J)
527 IF(NUIA) 705,706,706
705 WRITE(6,133)K1
      WRITE(6,133)((UR(I,J),J=1,KQ2),I=1,KQ2)
      WRITE(6,133)((UE(I,J),J=1,KQ2),I=1,KQ2)
967
968
976
706 DO 529 I=1,KQ2
      DO 529 J=1,KQ2
      C1=GR(I,J,K1)
      C2=GE(I,J,K1)
      DO 528 K=1,KQ2
      C1=C1+WA(I,K)*UR(K,J)-WB(I,K)*UE(K,J)
528 C2=C2+WA(I,K)*UE(K,J)+WB(I,K)*UR(K,J)
      WR(I,J)=C1
529 WE(I,J)=C2
      IF(K1=3) 550,522,522
522 IF(KC) 535,530,530
530 C1=WR(1,1)*WR(2,2)-WR(1,2)*WR(2,1)-WE(1,1)*WE(2,2)+WE(1,2)*WE(2,1)
      C2=WR(1,1)*WE(2,2)+WR(2,2)*WE(1,1)-WR(1,2)*WE(2,1)-WR(2,1)*WE(1,2)
      C3=C1+C2*C2
      C1=-C1/C3
      C2=-C2/C3
      EMR(1,1)=C1*WR(2,2)+C2*WE(2,2)
      EME(1,1)=C1*WE(2,2)-C2*WR(2,2)
      EWR(1,2)=-C1*WR(1,2)-C2*WE(1,2)
      EME(1,2)=C2*WR(1,2)-C1*WE(1,2)
      EWR(2,1)=-C1*WR(2,1)-C2*WE(2,1)

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      EME(2,1)=C2*WR(2,1)-C1*WE(2,1)
      EMR(2,2)=C1*WR(1,1)+C2*WE(1,1)
      EME(2,2)=C1*WE(1,1)-C2*WR(1,1)
      GO TO 546
  535 DO 536 I=1,4
      DO 536 J=1,4
      UR(I,J)=WR(I,J)
  536 UE(I,J)=WE(I,J)
      CALL MATINV(UR,4,WCC,0,DVN,1D)
      GO TO (538,537),1D
  537 WRITE(6,134)K1
      GO TO 553
  538 CALL MATINV(UE,4,WCC,0,DED,1D)
      GO TO (541,539),1D
  539 DO 540 I=1,4
      DO 540 J=1,4
      EMR(I,J)=-UR(I,J)
  540 EME(I,J)=0.0
      GO TO 546
  541 DO 543 I=1,4
      DO 543 J=1,4
      C1=0.0
      DO 542 K=1,4
  542 C1=C1+UR(I,K)*WE(K,J)+UE(I,K)*WR(K,J)
  543 AMR(I,J)=C1
      CALL MATINV(AMR,4,WCC,0,DVN,1D)
      GO TO (708,707),1C
  707 WRITE(6,139)K1
      GO TO 553
  708 DO 545 I=1,4
      DO 545 J=1,4
      C1=0.0
      C2=0.0
      DO 544 K=1,4
  544 C1=C1+AMR(I,K)*UE(K,J)
      C2=C2-AMR(I,K)*UR(K,J)
      EMR(I,J)=-C1
  545 EME(I,J)=-C2
  546 DO 534 I=1,KQ2
      DO 534 J=1,KQ2
      C1=0.0
      C2=0.0
      DO 531 K=1,KQ2
  531 C1=C1+EMR(I,K)*WA(K,J)+EME(I,K)*WB(K,J)
      C2=C2+EME(I,K)*WA(K,J)-EMR(I,K)*WB(K,J)
      IF(K2) 533,532,533
  532 AMR(I,J)=C1
      AME(I,J)=C2
      GO TO 534
  533 CMR(I,J)=C1
      CME(I,J)=C2
  534 CONTINUE
      IF(K1-3) 550,547,553
  547 DO 549 I=1,KQ2
      DO 549 J=1,KQ2
      C1=GR(I,J,1)

```

ONE

- EFN SOURCE STATEMENT - (EFNS) -

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```

548 DO 548 K=1,KQ2
548 C1=C1+2.0*WA(I,K)*AMR(I,J)-2.0*WB(I,K)*AME(K,J)
549 DEVN(I,J)=C1
      GO TO 553
550 K8=-1
      DO 552 I=1,KQ2
      K8=K8+2
      K9=-1
      DO 551 J=1,KQ2
      K9=K9+2
      DEDD(K8,K9)=WR(I,J)+WA(I,J)
      DEDD(K8,K9+1)=-WE(I,J)-WB(I,J)
      DEDD(K8+1,K9)=WE(I,J)-WB(I,J)
551 DEDD(K8+1,K9+1)=WR(I,J)-WA(I,J)
552 CONTINUE
553 CONTINUE
      IF(INDIA) 709,710,710
709 WRITE(6,140) 1160
      WRITE(6,133)((DEVN(I,J),J=1,KQ2),I=1,KQ2) 1161
      WRITE(6,141) 1171
      WRITE(6,133)((DED(I,J),J=1,KQ1),I=1,KQ1) 1172
710 CALL MATINV(DEVN,KQ2,WCC,0,DVN,1D) 1183
      GO TO (582,581),ID
581 DVN=0.0
      IJKL=1
      WRITE(6,136) IJKL 1186
582 CALL MATINV(DED,KQ1,WCC,0,DED,1D) 1188
      GO TO (584,583),ID
583 DED=0.0
      IJKL=2
      WRITE(6,136) IJKL 1191
584 WRITE(6,102) QX1,DVN,DED 1192
      IF(K7) 554,555,554
554 K7=0
      DVN1=DVN
      GO TO 561
555 IF(K6) 556,556,559
556 IF(K3) 565,557,564
557 IF(DVN=DVN1) 562,558,558
558 DVN1=DVN
559 IF(K4) 571,560,570
560 IF(DED=DED1) 566,561,561
561 DED1=DED
      Q1=QX1
      K5=0
      GO TO 510
562 K3=1
      DVN3=DVN
      OED3=DED
      D3=DVN
563 Q3=QX1
      QX1=0.5*(Q1+Q3)
      GO TO 515
564 K3=-1
      D1=DVN1
      D2=DVN

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LINES ONE

- SPN SOURCE STATEMENT - IFN(8) -

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562 DED=DED
Q2=QX1
GO TO 575
563 K3=0
K5=1
QX1=Q3
DVN=DVN3
DED=DED3
GO TO 558
566 K6=1
IF(K5) 569,569,567
567 K5=0
568 K4=-1
D1=DED1
D2=DED2
D3=DED3
GO TO 575
569 K4=1
D1,D3=DED
GO TO 563
570 Q2=QX1
DED2=DED
GO TO 568
571 K4=0
K6=0
QX1=Q3
DVN=DVN1
DED=DED3
GO TO 561
575 C1=Q2-Q1
C2=Q3-Q2
C3=Q3-Q1
C4=C2*C3
C3=C1*C3
C3=(D1-D2)/C3
C4=(D3-D2)/C4
C2=C1*C4-C2*C3
C1=C4*C3
IF(C1) 577,576,577
576 C5=-D2/C2
GO TO 580
577 C3=0.5*C2/C1
C4=SQR((C3+C3-D2/C1))
IF(C3) 579,578,578
578 C5=C4-C3
GO TO 580
579 C5=-C4-C3
580 QX1=Q2+C5
GO TO 515
599 SPD=SPD+SPIN
IF(SPN+0.000001-SPD) 600,600,260
600 NSP1=NSP1+1
IF(NSP-NSP1) 601,230,230
601 IF(INP) 602,190,602
602 STOP
99 FORMAT (1H1)

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ONE

- EFM SOURCE STATEMENT - (FMS)

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```
100 FORMAT(72H
1
101 FORMAT(12I5)
102 FORMAT(1P5E14.6)
103 FORMAT(118HSTATIONS BEARINGS MAGN.ST F+M/F/M RIG.PED NO.
1PADS COMPRESS HARMONICS NO.Q NO.SPEEDS DIAGNOS INPUT)
104 FORMAT(16,11I10)
105 FORMAT(42H YOUNGS MOD. DENSITY (SHAPE FACT)*G)
106 FORMAT(8E9.2)
107 FORMAT(15,1P16.6,1P7E14.6)
108 FORMAT(104H STATION MASS,LBS POLAR MOM.IN. TRANSV.MOM.IN L
LENGTH OUT.DIA(STIFF) OUT.DIA(MASS) INNER DIA.)
109 FORMAT(17HOBEARING STATIONS)
110 FORMAT(11HOROTOR DATA)
111 FORMAT(14HOCESTAL DATA)
112 FORMAT(89H STATION MASS-X,LBS STIFFNESS-X DAMPING-X MASS
1-Y,LBS STIFFNESS-Y DAMPING-Y)
113 FORMAT(20H)MAGNETIC FORCE DATA)
114 FORMAT(27H Q(0),FORCE Q(0),MOMENT)
115 FORMAT(28HOMATRIX OF COSINE COMPONENTS)
116 FORMAT(/13HOBEARING DATA)
117 FORMAT(19HOBEARING AT STATION,I3)
118 FORMAT(9H0HARMONIC4X3HXX10X5HW#BX10X3HKXY10X5HW#BX10X3HKYX10X5H
1W#BYX10X3HKYY10X5HW#BYY)
119 FORMAT(8HOPAD NO.,I3)
120 FORMAT(55H PITCH MOM.IN. PAD MASS PIVOT STIFFN. PIVOT ANGLE)
121 FORMAT(13HIROTOR SPEED=,1P13.6,4H RPM)
122 FORMAT(/13H0HARMONIC NO.,I3,11H,FREQUENCY=,1P13.6,8H RAD/SEC)
123 FORMAT(21HOBEARING COEFFICIENTS)
124 FORMAT(8H STATION5X3HKXX9X7HFRQ#BX9X3HKXY9X7HFRQ#BX9X3HKYX9X7HFR
1Q#BYX9X3HKYY9X7HFRQ#BYY)
125 FORMAT(31HOROTOR-BEARING IMPEDANCE MATRIX)
126 FORMAT(26HOMATRIX OF SINE COMPONENTS)
127 FORMAT(10HREAL PART)
128 FORMAT(15H0IMAGINARY PART)
129 FORMAT(11,9X3HQXX7X25HEVEN DETERM. ODD DETERM.)
130 FORMAT(34H0REAL PART OF A-MATRIX IS SINGULAR)
131 FORMAT(10H HARMONIC=,I3,12H FREQUENCY=,1P13.6)
132 FORMAT(42H0REAL PART OF IMPEDANCE MATRIX IS SINGULAR)
133 FORMAT(1P4E14.6)
134 FORMAT(37H0REAL PART OF S-MATRIX IS SINGULAR,K=,I3)
135 FORMAT(18H0S-MATRIX FOR K=1=,I3)
136 FORMAT(20H0DETERMINANT ZERO AT,I3)
137 FORMAT(35H0INVERSION MATRIX FOR A IS SINGULAR)
138 FORMAT(35H0INVERSION MATRIX FOR E IS SINGULAR)
139 FORMAT(38H0INVERSION MATRIX FOR S IS SINGULAR,K=I3)
140 FORMAT(17H0EVEN DETERMINANT)
141 FORMAT(16H0ODD DETERMINANT)
END
```

DAVE - EPM SOURCE STATEMENT - IFN(S) -

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C	MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS	MAT1	2
C	NOVEMBER 1992 S GOOD DAVID TAYLOR MODEL BASIN AN RATTI	MAT1	3
C	SUBROUTINE MATINV(A,M1,B,M1,DETERM,IDI)	MAT1	4
C	GENERAL FORM OF DIMENSION STATEMENT.	MAT1	5
C	DIMENSION A(8,8),B(8,1)	MAT1	6
C	DIMENSION INDEX(8,3)	MAT1	7
C	EQUIVALENCE (IROW,JROW), (ICOLUMN,JCOLUMN), (AMAX, TA, SWAP)	MAT1	8
C	INITIALIZATION	MAT1	11
C	M=M1	MAT1	12
C	N=N1	MAT1	13
C	DO 6 I=1,N	MAT1	14
C	K1=1	MAT1	15
C	K2=1	MAT1	16
C	DO 6 J=1,N		
C	IF(A(I,J)) .3.4.3.		
3	K2=0		
4	IF(A(J,1)) .5.6.5		
5	K2=0		
6	CONTINUE		
C	IF(K1+K2) 6,8,7		
7	ID=2		
C	DETERM=9.0		
C	GO TO 740		
C	8 CONTINUE		
10	DETERM=1.0		
19	DO 20 J=1,N		
20	INDEX(J,3)=0		
30	DO 550 I=1,N		
C	SEARCH FOR PIVOT ELEMENT		
C	40 AMAX=0.0		
45	DO 105 I=1,N		
C	IF(INDEX(I,3)-1) 60, 105, 60		
60	DO 100 K=1,N		
C	IF(INDEX(K,3)-1) 80, 100, 715		
80	IF(AMAX-ABS(A(I,K))) .85.100.100		
85	IROW=J		
90	ICOLUMN=K		
C	AMAX=ABS(A(I,K))		
100	CONTINUE		
105	CONTINUE		
C	INDEX(ICOLUMN,3) = INDEX(ICOLUMN,3) +1		
260	INDEX(I,1)=IROW		
270	INDEX(I,2)=ICOLUMN		
C	INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL		
C	130 IF (IROW-ICOLUMN) 140, 310, 140		
140	DETERM=DETERM		

DAVE EPN SOURCE STATEMENT - IFN(S) -

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150 DO 200 L=1,N	MAT1 43
160 SWAP=A(IROW,L)	MAT1 44
170 A(IROW,L)=A(ICOLUMN,L)	MAT1 45
200 A(ICOLUMN,L)=SWAP	MAT1 46
IF(N) 310, 310, 210	MAT1 47
210 DO 250 L=1, M	MAT1 48
220 SWAP=B(IROW,L)	MAT1 49
230 B(IROW,L)=B(ICOLUMN,L)	MAT1 50
250 B(ICOLUMN,L)=SWAP	MAT1 51
C	
C DIVIDE PIVOT ROW BY PIVOT ELEMENT	
C	
310 PIVOT =A(ICOLUMN,ICOLUMN)	MAT1 52
DETERM=DETERM*PIVOT	MAT1 53
330 A(ICOLUMN,ICOLUMN)=1.0	MAT1 54
340 DO 350 L=1,N	MAT1 55
350 A(ICOLUMN,L)=A(ICOLUMN,L)/PIVOT	MAT1 56
355 IF(N) 380, 380, 360	MAT1 57
360 DO 370 L=1,M	MAT1 58
370 B(ICOLUMN,L)=B(ICOLUMN,L)/PIVOT	MAT1 59
C	
C REDUCE NON-PIVOT ROWS	
C	
380 DO 550 L1=1,N	MAT1 60
390 IF(L1-ICOLUMN) 400, 550, 400	MAT1 61
400 T=A(L1,ICOLUMN)	MAT1 62
420 A(L1,ICOLUMN)=0.0	MAT1 63
430 DO 450 L=1,N	MAT1 64
450 A(L1,L)=A(L1,L)-A(ICOLUMN,L)*T	MAT1 65
455 IF(M) 550, 550, 460	MAT1 66
460 DO 500 L=1,M	MAT1 67
500 B(L1,L)=B(L1,L)-B(ICOLUMN,L)*T	MAT1 68
550 CONTINUE	MAT1 69
G	
G INTERCHANGE COLUMNS	
C	
600 DO 710 I=1,N	MAT1 70
610 L=N+1-I	MAT1 71
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630	MAT1 72
630 JROW=INDEX(L,1)	MAT1 73
640 JCOLUMN=INDEX(L,2)	MAT1 74
650 DO 705 K=1,N	MAT1 75
660 SWAP=A(K,JROW)	MAT1 76
670 A(K,JROW)=A(K,JCOLUMN)	MAT1 77
700 A(K,JCOLUMN)=SWAP	MAT1 78
705 CONTINUE	MAT1 79
710 CONTINUE	MAT1 80
DO 730 K = 1,N	MAT1 81
IF(INDEX(K,3) -1) 715,720,715	MAT1 82
715 ID =2	MAT1 83
GO TO 740	MAT1 84
720 CONTINUE	MAT1 85
730 CONTINUE	MAT1 86
ID=1	MAT1 87
740 RETURN	MAT1 88
E'0	MAT1 89
	MAT1 90
	MAT1 91
	MAT1 92
	MAT1 93
	MAT1 94
	MAT1 95
	MAT1 96
	MAT1 97
	MAT1 98
	MAT1 99

4-11-1967

11	2	7	-1	0	0	1	10	0	1	0	1
300000000.0				0.283			8630000.0				
6.15	62.4	31.0	1.3		3.5		3.5		3.339		
1.61	0.0	0.0	1.45		3.5		3.6		3.2		
0.0	0.0	0.0	2.37		3.5		3.65		3.062		
0.0	0.0	0.0	4.83		3.5		3.65		3.062		
0.0	0.0	0.0	1.67		2.775		3.25		2.0		
3.1	12.0	12.0	2.1		3.5		3.7		1.04		
0.0	0.0	0.0	2.1		3.5		3.7		1.04		
3.1	12.0	12.0	1.67		2.775		3.25		2.0		
0.0	0.0	0.0	3.49		3.5		3.65		3.062		
0.0	0.0	0.0	4.39		3.5		3.65		3.062		
9.21	56.0	28.0	0.0		1.0		0.0		0.0		
4	10										
10300.0		20300.0			1000.0		2.0		1.0		
20000.0		88200.0									
0.0	0.0	4.2			0.0						
0.0	0.0	0.0			-4.2						
4.2	0.0	0.0			0.0						
0.0	-4.2	0.0			0.0						
0.0	0.0	0.0			-4.2						
0.0	-4.2	0.0			0.0						
-4.2	0.0	0.0			0.0						
0.0	1000000.0	10000.0									
111915.8	0.0	0.0	0.0		0.0		0.0		111915.8	0.0	
196262.3	0.0	0.0	0.0		0.0		0.0		196262.3	0.0	
197298.6	0.0	0.0	0.0		0.0		0.0		197298.6	0.0	
184631.7	0.0	0.0	0.0		0.0		0.0		184631.7	0.0	
168424.7	0.0	0.0	0.0		0.0		0.0		168424.7	0.0	
126272.4	0.0	0.0	0.0		0.0		0.0		126272.4	0.0	
254752.4	0.0	0.0	0.0		0.0		0.0		254752.4	0.0	
217609.4	0.0	0.0	0.0		0.0		0.0		217609.4	0.0	
181060.1	0.0	0.0	0.0		0.0		0.0		181060.1	0.0	
121742.5	0.0	0.0	0.0		0.0		0.0		121742.5	0.0	
-349724.4	0.0	0.0	0.0		0.0		0.0		-349724.4	0.0	
111915.8	0.0	0.0	0.0		0.0		0.0		111915.8	0.0	
196262.3	0.0	0.0	0.0		0.0		0.0		196262.3	0.0	
197298.6	0.0	0.0	0.0		0.0		0.0		197298.6	0.0	
184631.7	0.0	0.0	0.0		0.0		0.0		184631.7	0.0	
168424.7	0.0	0.0	0.0		0.0		0.0		168424.7	0.0	
126272.4	0.0	0.0	0.0		0.0		0.0		126272.4	0.0	
254752.4	0.0	0.0	0.0		0.0		0.0		254752.4	0.0	
217609.4	0.0	0.0	0.0		0.0		0.0		217609.4	0.0	
181060.1	0.0	0.0	0.0		0.0		0.0		181060.1	0.0	
121742.5	0.0	0.0	0.0		0.0		0.0		121742.5	0.0	
-349724.4	0.0	0.0	0.0		0.0		0.0		-349724.4	0.0	

TURBINE DRIVEY 4 POLE ALTERNATOR, STABILITY CHECK 4-11-1967

STATION	BEARINGS	MAGN. ST	F4H/F/H	RIG.PED	NO.PADS	COMPRESS	HARMONICS	NO.Q	NO.SPEEDS	DIAGNOS	INPUT
11	2	7	-1	0	0	1	10	0	1	0	0

YOUNGS MOD. DENSITY (SHAPE FACT)*6
 3.03303E 07 2.83000E-01 R.630000E 06

ROTOR DATA

STATION	MASS, LBS	POLAR MOM.	IN.	TRANSV. MOM.	IN.	LENGTH	OUT. DIA(STIFF)	OUT. DIA(MASS)	INNER DIA.
1	6.15000E 03	6.24000E 01	3.10000E 01	1.30000E 00	3.50000E 00	3.50000E 00	3.339000E 00	3.50000E 00	3.339000E 00
2	1.61000E 03	0.	0.	1.45000E 00	3.10000E 00	3.60000E 00	3.20000E 00	3.60000E 00	3.20000E 00
3	0.	0.	0.	2.37000E 00	3.20000E 00	3.65000E 00	3.062000E 00	3.65000E 00	3.062000E 00
4	0.	0.	0.	6.83000E 00	3.50000E 00	3.65000E 00	3.062000E 00	3.65000E 00	3.062000E 00
5	0.	0.	0.	1.67000E 00	2.77500E 00	3.25000E 00	2.60000E 00	3.25000E 00	2.60000E 00
6	3.10000E 00	1.20000E 01	1.20000E 01	2.10000E 00	3.50000E 00	3.70000E 00	1.040000E 00	3.50000E 00	3.70000E 00
7	0.	0.	0.	2.10000E 00	3.50000E 00	3.70000E 00	1.040000E 00	3.50000E 00	3.70000E 00
8	3.10000E 00	1.20000E 01	1.20000E 01	1.67000E 00	2.77500E 03	3.25000E 00	2.00000E 00	3.25000E 00	3.25000E 00
9	0.	0.	0.	3.49000E 00	3.50000E 00	3.65000E 00	3.062000E 00	3.50000E 00	3.062000E 00
10	0.	0.	0.	4.39000E 00	3.50000E 00	3.65000E 00	3.062000E 00	3.50000E 00	3.062000E 00
11	9.21000E 00	5.60000E 01	2.90000E 01	0.	1.05000E 00	0.	0.	0.	0.

BEARING STATIONS
4 10

MAGNETIC FORCE DATA

Q(0), FORCE Q(0), MOMENT
 2.00000E 04 8.82000E 04

MATRIX OF COSINE COMPONENTS

0.	0.	4.20000E 00	0.
0.	0.	0.	-4.20000E 00
4.20000E 00	0.	0.	0.
0.	-4.20000E 00	0.	0.

MATRIX OF SINE COMPONENTS

0.	0.	0.	-4.20000E 00
0.	0.	-4.20000E 00	0.
0.	-4.20000E 00	0.	0.
-4.20000E 00	0.	0.	0.

BEARING DATA

BEARING AT STATION 4

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYY	KYY	W*BYY
0	1.119158E 05	0.	0.	0.	0.	1.119158E 05	0.	0.
1	1.962623E 05	0.	0.	0.	0.	1.962623E 05	0.	0.
2	1.972986E 05	0.	0.	0.	0.	1.972986E 05	0.	0.
3	1.846317E 05	0.	0.	0.	0.	1.846317E 05	0.	0.
4	1.684247E 05	0.	0.	0.	0.	1.684247E 05	0.	0.
5	1.262724E 05	0.	0.	0.	0.	1.262724E 05	0.	0.
6	2.547524E 05	0.	0.	0.	0.	2.547524E 05	0.	0.
7	2.176094E 05	0.	0.	0.	0.	2.176094E 05	0.	0.
8	1.810601E 05	0.	0.	0.	0.	1.810601E 05	0.	0.
9	1.217425E 05	0.	0.	0.	0.	1.217425E 05	0.	0.
10	-3.497244E 05	0.	0.	0.	0.	-3.497244E 05	0.	0.

BEARING AT STATION 10

HARMONIC	KXX	W*BXX	KXY	W*BXY	KYX	W*BYY	KYY	W*BYY
0	1.119158E 05	0.	0.	0.	0.	1.119158E 05	0.	0.
1	1.962623E 05	0.	0.	0.	0.	1.962623E 05	0.	0.
2	1.972986E 05	0.	0.	0.	0.	1.972986E 05	0.	0.
3	1.846317E 05	0.	0.	0.	0.	1.846317E 05	0.	0.
4	1.684247E 05	0.	0.	0.	0.	1.684247E 05	0.	0.
5	1.262724E 05	0.	0.	0.	0.	1.262724E 05	0.	0.
6	2.547524E 05	0.	0.	0.	0.	2.547524E 05	0.	0.
7	2.176094E 05	0.	0.	0.	0.	2.176094E 05	0.	0.
8	1.810601E 05	0.	0.	0.	0.	1.810601E 05	0.	0.
9	1.217425E 05	0.	0.	0.	0.	1.217425E 05	0.	0.
10	-3.497244E 05	0.	0.	0.	0.	-3.497244E 05	0.	0.

ROTOR SPEED= 1.030000E 04 RPM

UNDERFLOW AT 53363 IN HQ

CXX	EVEN DETERM.	ODD DETERM.
-0.	2.111015E 00	1.915474E 00
1.000000E 04	2.114852E 00	1.912247E 00
2.000000E 04	2.126397E 00	1.902599E 00
3.000000E 04	2.145743E 00	1.4866603E 00
4.000000E 04	2.173049E 00	1.464410E 00
5.000000E 04	2.208537E 00	1.436213E 00
6.000000E 04	2.252498E 00	1.402247E 00
7.000000E 04	2.305294E 00	1.362842E 00
8.000000E 04	2.367357E 00	1.318312E 00
9.000000E 04	2.439197E 00	1.269057E 00
1.000000E 05	2.521408E 00	1.215507E 00
1.100000E 05	2.614667E 00	1.158129E 00
1.200000E 05	2.719745E 00	1.097423E 00
1.300000E 05	2.837510E 00	1.033916E 00
1.400000E 05	2.968945E 00	9.681555E-01
1.500000E 05	3.115132E 00	9.007096E-01
1.600000E 05	3.277292E 00	8.321563E-01
1.700000E 05	3.456775E 00	7.630799E-01
1.800000E 05	3.655079E 00	6.940651E-01
1.900000E 05	3.873851E 00	6.256909E-01
2.000000E 05	4.114905E 00	5.585233E-01
2.100000E 05	4.380256E 00	4.931183E-01
2.200000E 05	4.672125E 00	4.299975E-01
2.300000E 05	4.992890E 00	3.696616E-01
2.400000E 05	5.345258E 00	3.125751E-01
2.500000E 05	5.732106E 00	2.591635E-01
2.600000E 05	6.156636E 00	2.098082E-01
2.700000E 05	6.622316E 00	1.648418E-01
2.800000E 05	7.132986E 00	1.245444E-01
2.900000E 05	7.692774E 00	8.913947E-02
3.000000E 05	8.306188E 00	5.879141E-02
3.100000E 05	8.978231E 00	3.360245E-02
3.200000E 05	9.714280E 00	1.361092E-02
3.300000E 05	1.052014E 01	-1.210047E-03
3.250000E 05	1.010808E 01	5.558361E-03
3.290321E 05	1.043891E 01	5.862245E-07
3.400000E 05	1.140215E 01	-1.095307E-02
3.500000E 05	1.236721E 01	-1.577639E-02
3.600000E 05	1.342293E 01	-1.590296E-02
3.700000E 05	1.457748E 01	-1.161866E-02
3.800000E 05	1.583943E 01	-3.269779E-03
3.900000E 05	1.721817E 01	6.740358E-03
3.850000E 05	1.651351E 01	2.304956E-03
3.830249E 05	1.62436CE 01	-3.156500E-06
4.000000E 05	1.87245CE 01	2.395571E-02
4.100000E 05	2.037091E 01	4.187267E-02
4.200000E 05	2.216305E 01	6.194603E-02
4.300000E 05	2.412313E 01	8.359595E-02
4.400000E 05	2.626082E 01	1.062157E-01
4.500000E 05	2.859093E 01	1.291802E-01
4.600000E 05	3.112933E 01	1.518559E-01
4.700000E 05	3.389487E 01	1.736104E-01
4.800000E 05	3.690608E 01	1.938243E-01
4.900000E 05	4.018274E 01	2.119022E-01
5.000000E 05	4.374745E 01	2.272860E-01
5.100000E 05	4.762281E 01	2.394674E-01
5.200000E 05	5.183484E 01	2.480018E-01

5.300000E 05	5.640991E 01	2.525327E-01
5.400000E 05	6.137715E 01	2.527559E-01
5.500000E 05	6.676704E 01	2.485348E-01
5.600000E 05	7.261270E 01	2.398153E-01
5.700000E 05	7.894963E 01	2.266910E-01
5.800000E 05	8.581567E 01	2.094089E-01
5.900000E 05	9.325071E 01	1.883841E-01
6.000000E 05	1.012980E 02	1.642147E-01
6.100000E 05	1.100037E 02	1.376969E-01
6.200000E 05	1.194158E 02	1.098385E-01
6.300000E 05	1.295874E 02	8.187246E-02
6.400000E 05	1.405734E 02	5.526939E-02
6.500000E 05	1.524326E 02	3.174912E-02
6.600000E 05	1.652282E 02	1.329130E-02
6.700000E 05	1.790253E 02	2.144265E-03
6.800000E 05	1.938964E 02	8.327640E-04
6.900000E 05	2.099157E 02	1.216391E-02
7.000000F 05	2.271629E 02	3.923128E-02
7.100000E 05	2.457224E 02	8.541692E-02
7.200000E 05	2.656836E 02	1.543913E-01
7.300000E 05	2.871419E 02	2.501109E-01
7.400000E 05	3.101962E 02	3.768126E-01
7.500000E 05	3.349534E 02	5.390065E-01
7.600000E 05	3.615265E 02	7.414649E-01
7.700000E 05	3.900315E 02	9.892078E-01
7.800000E 05	4.205948E 02	1.287487E 00
7.900000E 05	4.533498E 02	1.641763E 00
8.000000E 05	4.884344E 02	2.057687E 00
8.100000E 05	5.259974E 02	2.541064E 00
8.200000E 05	5.661915E 02	3.097830E 00
8.300000E 05	6.091827E 02	3.734009E 00
8.400000E 05	6.551415E 02	4.455679E 00
8.500000E 05	7.042479E 02	5.268929E 00
8.600000E 05	7.566958E 02	6.179804E 00
8.700000E 05	8.126833E 02	7.194261E 00
8.800000E 05	8.724218E 02	8.318111E 00
8.900000E 05	9.361331E 02	9.556956E 00
9.000000E 05	1.004051E 03	1.091613E 01
9.100000E 05	1.076420E 03	1.240063E 01
9.200000E 05	1.153493E 03	1.401503E 01
9.300000E 05	1.235545E 03	1.576344E 01
9.400000E 05	1.322856E 03	1.764940E 01
9.500000E 05	1.415717E 03	1.967578E 01
9.600000E 05	1.514448E 03	2.184475E 01
9.700000E 05	1.619368E 03	2.415767E 01
9.800000E 05	1.730818E 03	2.661496E 01
9.900000E 05	1.849154E 03	2.921608E 01

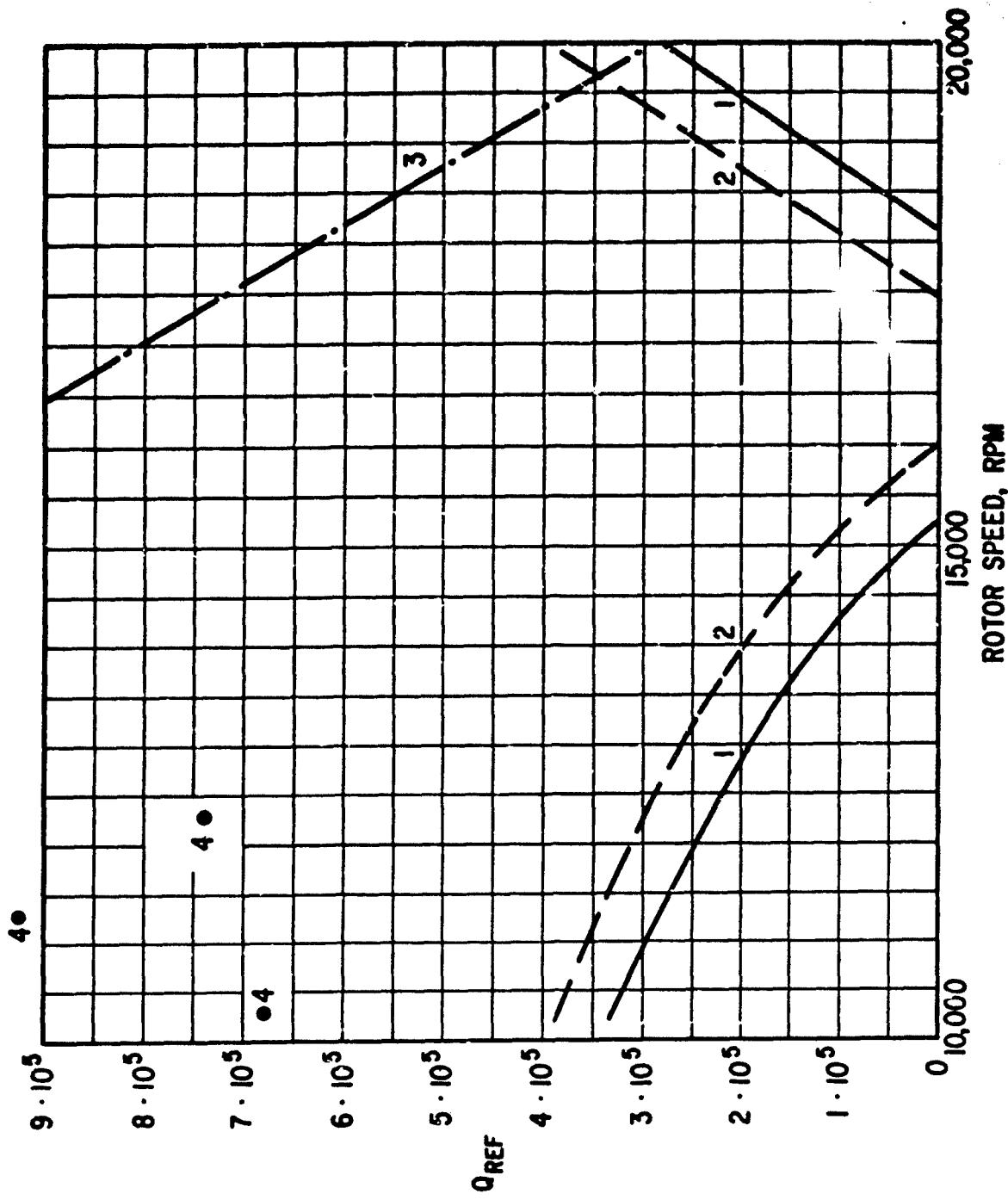


Figure 20 Stability Map for Four Pole Humpolar Generator

APPENDIX XII: Computer Program - The Response of a Rotor with Timevarying Magnetic Forces

This appendix describes the computer program PN0354: "The Response of a Rotor with Timevarying Magnetic Forces" and gives the detailed instructions for using the program. The program is based on the analysis contained in Appendix X (and Appendix VIII). It calculates the whirl amplitude of an alternator rotor which is eccentric and misaligned with respect to the axis of the alternator stator.

The response program has most features in common with the stability program. Both programs employ the same model of the rotor-bearing system and the form of the generator magnetic forces is the same for the two programs. Hence, much of the input data is the same for the two programs and in giving the instructions for preparation of the input to the response program, reference will be made to the instructions already given for the stability program in Appendix XI.

COMPUTER INPUT

An input data form is given in back of this appendix for quick reference when preparing the computer input data. In the following the more detailed instructions are given except for those parts which have already been covered in Appendix XI.

Card 1 (72^W) Any descriptive text may be given, identifying the calculation.

Card 2 (1115) This is the "Control" cards whose values control the rest of the input. It is identical to card 2 of the stability program with a few exceptions, the major one being that the previous item 3, NQ, is eliminated. Card 2 has 11 values:

- 1.NS specifies the number of rotor stations ($NS \leq 50$)
- 2.NB specifies the number of bearings ($1 \leq NB \leq 10$)
- 3.KA see Appendix XI
- 4.KC see Appendix XI

5.NRP see Appendix XI

6.NPD see Appendix XI

7.INC see Appendix XI

8.NH In the response calculation, the program evaluates the frequency response of the rotor-bearing system at certain discrete frequencies. These frequencies are the harmonics of the magnetic force frequency, i.e. at $0,\Omega, 2\Omega, 3\Omega, 4\Omega, \dots$, where Ω is the magnetic force frequency (in the stability calculation the frequencies are $0,\frac{1}{2}\Omega, \frac{3}{2}\Omega, \frac{5}{2}\Omega, \frac{7}{2}\Omega, \dots$). NH specifies the number of the highest harmonic such that the highest frequency is $(NH)\cdot\Omega$. NH must be equal to or greater than 1 but it cannot exceed 10.

9.NEP see Appendix XI, card 2, item 10

10.NDIA see Appendix XI, card 2, Item 11

11.INP see Appendix XI, card 2, item 12

Card 3 (1P5E14.6)

See Appendix XI

Rotor Data (8E9.2)

See Appendix XI

Bearing Stations (11I5)

See Appendix XI

Pedestal Data (8E9.2)

See Appendix XI

Whirl Orbit Points in Output (1P5E14.6)

The rotor amplitudes x and y are calculated for each rotor station in the form:

$$x = \sum_{k=0}^{NH} [x_{ck} \cos(k\Omega t) - x_{sk} \sin(k\Omega t)] \quad (M.1)$$

$$y = \sum_{k=0}^{NH} [y_{ck} \cos(k\Omega t) - y_{sk} \sin(k\Omega t)] \quad (M.2)$$

where Ω is the frequency of the magnetic forces in radians/sec, t is time in seconds and NH is item 8 on card 2. The program output lists x_{ck} , y_{ck} , u_{ck} and v_{ck} for $0 \leq k \leq NH$. However, in order to get the maximum amplitude it is necessary to calculate the whirl orbit whose coordinates, of course, are x and y from eqs. (M.1) and (M.2). This is done by calculating x and y at discrete points along the orbit. Let ω be the angular speed of the rotor. Then:

$$k\Omega t = k\left(\frac{\Omega}{\omega}\right)(\omega t) \quad \underline{0 \leq k \leq NH} \quad (M.3)$$

where $\left(\frac{\Omega}{\omega}\right)$ is the fixed ratio between the magnetic force frequency and the speed of the rotor (see the following input card). By varying (ωt) from 0 to 360 degrees, the complete whirl orbit for one shaft revolution can be obtained. The present input card specifies what range of (ωt) is desired and in how big increments the range should be carried. The card has four values:

1. Initial value of (ωt) , degrees
2. Final value of (ωt) , degrees
3. Increment of (ωt) , degrees
4. The lower limit of amplitude values of interest in inches. This item is included because it frequently happens that the amplitudes of the higher harmonics are very small. Then there is no need to include them in the output. This input item specifies what is the smallest amplitude value of interest which may be, say, 10^{-6} inches (1 microinch).

Speed Data (1P5E14.6)

See Appendix XI

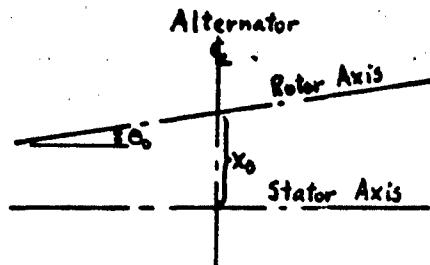
Magnetic Force Data

See Appendix XI but disregard all references to Q_{ref} .

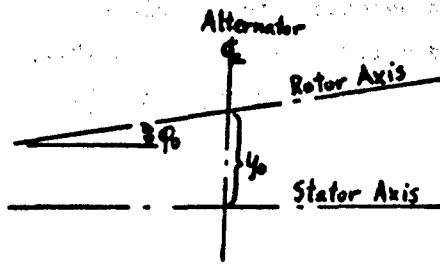
Rotor Eccentricity and Misalignment Coordinates (1P4E14.6)

The forces and moments that cause the rotor to whirl, only act when the rotor axis does not coincide with the axis of the alternator stator. Four

coordinates are needed to define the position of the rotor axis with respect to the alternator axis:



x-plane



y-plane

Here, x_e and y_e are the two eccentricity components in the centerplane of the alternator, and θ_e and ϕ_e are the two corresponding misalignment angles. These values are given on a card which follows next after the magnetic force data. If KC=1 (card 2, item 4), all four values must be specified:

1. Eccentricity, x_e , of rotor in the centerplane of the alternator, x-direction, inches.
2. Eccentricity, y_e , of rotor in the centerplane of the alternator, y-direction, inches.
3. Misalignment angle, θ_e , of rotor axis with respect to stator axis in the x-plane, radians or inches/inch.
4. Misalignment angle, ϕ_e , of rotor axis with respect to stator axis in the y-plane, radians or inches-inch.

If KC ≥ 0 , only two values can be specified. There will be one card with two values and one of the following two possibilities apply.

KC=0

1. x_e , inch
2. y_e , inch

KC=1

1. θ_e , radians
2. ϕ_e , radians.

Bearing Data, Fixed Geometry

This data applies when NPD=0 (item 6, card 2). The instructions for preparing the input are the same as previously given in Appendix XI. However, eq. (L.3) should be changed to read:

$$\frac{\omega}{\Omega} = 0, \left(\frac{\Omega}{\omega}\right), 2\left(\frac{\Omega}{\omega}\right), 3\left(\frac{\Omega}{\omega}\right), \dots, (NH)\left(\frac{\Omega}{\omega}\right) \quad (M.4)$$

This does not affect the number or the form of the input cards but only redefines those frequencies at which the bearing coefficients must be evaluated when the lubricant is compressible (INC=1, item 7, card 2).

Bearing Data, Tilting Pad Bearing

This data applies when NPD ≠ 0 (card 2, item 6). The instructions for preparing the input are the same as previously given in Appendix XI. However, when the lubricant is compressible (INC=1, item 7, card 2) those frequencies at which the pad film coefficients are evaluated, are given by eq. (M.4), not eq. (L.3).

COMPUTER OUTPUT

Referring to the later given sample calculation, it is seen that the first three pages of the computer output repeats the input data in the same order in which it is read in by the computer. The only input data which is not repeated in the output, is the card with the speed data and the card specifying the points on the whirl orbit.

Next follow the results of the calculations. First, the rotor speed is identified and, immediately after, the calculated rotor amplitudes and slopes at the centerplane of the alternator are given. There are 10 columns. The first column identifies the harmonic of the oscillation (i.e. 0, 1, 2,...times the frequency of the magnetic forces), and in the second column are the corresponding frequencies. The 8 remaining columns give the cosine and sine components of the two amplitudes (x and y) and the two slopes ($\Theta = \frac{dx}{dt}$ and $\Phi = \frac{dy}{dt}$). The resulting motion is obtained from eqs. (M1) and (M2) and the analogous ones for Θ and Φ . Here, NH is the number of the highest harmonic (in the sample case, NH=5), Ω is the magnetic force frequency, radians/sec, t is time in seconds, and k is the number of the harmonic, given in Column 1. In the output

$X(C) = x_{ck}$, inch, $X(S) = x_{sk}$, inch, $Y(C) = y_{ck}$, inch, $Y(S) = y_{sk}$, inch, $DX/DZ(C) = \theta_{ck}$, inch/inch, $DX/DZ(S) = \theta_{sk}$, inch/inch, $DY/DZ(C) = \phi_{ck}$, inch/inch, and $DY/DZ(S) = \phi_{sk}$, inch/inch. These data are given since they result from the most important part of the calculation, namely the solution of the simultaneous equations given by eq. (K.23), Appendix X.

Thereafter there is one page of output for each rotor station. The rotor station is identified first, followed by a list similar to the one described above except that no data are given for θ and ϕ . Next, eqs. (M.1) and (M.2) are used to calculate the rotor motion and the results are given in a five column list under the title "WHIRL ORBIT." For this purpose, eqs. (M.1) and (M.2) are rewritten as:

$$x = \sum_{k=0}^{NM} [x_{ck} \cos(k(\frac{\Omega}{\omega})\omega t) - x_{sk} \sin(k(\frac{\Omega}{\omega})\omega t)] \quad (M.5)$$

$$y = \sum_{k=0}^{NM} [y_{ck} \cos(k(\frac{\Omega}{\omega})\omega t) - y_{sk} \sin(k(\frac{\Omega}{\omega})\omega t)]$$

where ω is the angular speed of the rotor. With $(\frac{\Omega}{\omega})$ being a fixed ratio, ωt is varied over a range as specified in the input. The value of ωt gives the angle of rotation of the shaft such that as ωt goes from 0 to 360 degrees, the shaft makes one revolution. The first column, titled "SHAFT ROTAT, DEG", specifies the value of ωt . For a given value of ωt , eq. (M.5) can be used directly to calculate the amplitude components x and y , and the values are given in the output in the columns entitled "X" and "Y". They are in inches. Hence, x and y are simply the coordinates of the whirl orbit described by the center of the shaft during one revolution of the shaft. The output also gives the coordinates of the whirl orbit in polar coordinates in the two last columns where:

$$\text{"AMPLITUDE"} = \sqrt{x^2 + y^2} \quad \text{inches}$$

$$\text{"ANGLE X-AMPL"} = \tan^{-1}(y/x) \quad \text{degrees}$$

Thus the amplitude gives the "radius" of the orbit, and the angle is the angle from the x-axis to the amplitude direction, positive in the direction of rotor rotation.

SAMPLE CALCULATION

The response of a 4 pole homopolar alternator has been calculated to illustrate the use of the program. The rotor is supported in two gas lubricated tilting pad bearings, and the bearings have four shoes with the static load passing between the pivots of the two bottom shoes. In the centerplane of the alternator (rotor station 7), the center of the rotor coincides with the center of the alternator but the rotor axis is misaligned 0.002 inches/inch with respect to the magnetic axis of the alternator which results in a pulsating force of 84 lbs at a frequency of twice the shaft speed acting on the rotor. With the two first critical speeds at approximately 14,700 rpm and 16,000 rpm it is found that the maximum response occurs at half these speeds. However, only the fundamental harmonic is excited with a significant amplitude and the amplitudes of the higher harmonics are of no practical interest.

INPUT FORM FOR COMPUTER PROGRAM

PN0354: THE RESPONSE OF A ROTOR WITH TIMEVARYING MAGNETIC FORCES

Card 1 (72B)

Text

Card 2 (11I5)

1. NS = Number of rotor stations ($NS \leq 50$)
2. NB = Number of bearings ($NB \leq 10$)
3. KA. $|KA|$ = Rotor station number at which magnetic forces act
 - $KA > 0$: forces only, no moments }
 $KA < 0$: moments only, no forces }
 - $KC \geq 0$
 - $KA > 0$, $KC=-1$: both forces and moments
4. KC $KC=0$: the magnetic forces or moments are proportional to amplitudes
 $KC=1$: the magnetic forces or moments are proportional to slope
 $KC=-1$: there are both magnetic forces and moments
5. NRP $NRP=0$: bearing pedestals are rigid, no pedestal input data
 $NRP=1$: flexible bearing pedestals, pedestal input data required
6. NPD $NPD=0$: fixed geometry bearings
 $NPD \geq 1$: number of pads in tilting pad bearing, load between pads
 $NPD \leq -1$: $|NPD|$ = number of pads in tilting pad bearing, load on pad
7. INC $INC=0$: bearing lubricant is incompressible
 $INC=1$: bearing lubricant is compressible
8. NH = Number of frequency harmonics in stability calculation ($1 \leq NH \leq 10$)
9. NSP = Number of speed ranges with accompanying data ($NSP \geq 1$)
10. NDIA $NDIA=0$: rotor impedance matrices not included in output
 $NDIA=1$: rotor impedance matrices included in output
 $NDIA=-1$: diagnostic
11. INP $INP=0$: more input follows, starting from card 1
 $INP=1$: last set of input data

Card 3 (1P5E14.6)

1. YM = Youngs modulus for shaft material, lbs/in²
2. DNST = Weight density of shaft material, lbs/in³
3. SHM = αG , where G is shear modulus, lbs/in², and α is shape factor for shear.

Rotor Data (8E9.2)

Give NS cards with 7 numbers on each card:

1. Mass at rotor station, lbs.
2. Polar mass moment of inertia at rotor station, lbs-in²
3. Transverse mass moment of inertia at rotor station, lbs-in²
4. Length of shaft section to next station, inch
5. Outer shaft diameter for cross-sectional moment of inertia, inch
6. Outer shaft diameter for shaft mass, inch
7. Inner shaft diameter, inch.

Bearing Stations (11I5)

List the rotor stations at which there are bearings, in total NB stations

Pedestal Data (8E9.2)

This data only applies when NRP=1 (card 2, item 5). Give a total of NB cards with 6 values per card:

1. Pedestal mass, x-direction, lbs.
2. Pedestal stiffness, x-direction, lbs/inch
3. Pedestal damping, x-direction, lbs-sec/inch
4. Pedestal mass, y-direction, lbs.
5. Pedestal stiffness, y-direction, lbs/inch
6. Pedestal damping,y-direction, lbs-sec/inch

Whirl Orbit Points in Output (1P5E14.6)

Give one card with 4 values:

1. Initial value of ω_t , degrees
2. Final value of ω_t , degrees
3. Increment of ω_t , degrees
4. Lower limit for amplitude values of interest, inch

Note: The following data must be repeated NSP-times (Card 2, item 9)

Speed Data (1P5E14.6)

Give one card with 5 values:

1. Initial speed, rpm
2. Final speed, rpm
3. Speed increment, rpm
4. Ratio of magnetic force frequency to rotor speed
5. Scale factor for determinant (set equal to mass of rotor)

Magnetic Force Data

Card (1P5E14.6)

1. Static gradient of magnetic force, Q_0 , lbs/in
2. Static gradient of magnetic moment, Q'_0 , lbs-inch/radian

Cards (1P4E14.6)

a. If XC=-1 (card 2, item 4), give 8 cards with 4 values per card:

Q_{xx}	Q_{xy}	$Q_{x\theta}$	$Q_{x\varphi}$
Q_{yx}	Q_{yy}	$Q_{y\theta}$	$Q_{y\varphi}$
$Q_{\theta x}$	$Q_{\theta y}$	$Q_{\theta\theta}$	$Q_{\theta\varphi}$
$Q_{\varphi x}$	$Q_{\varphi y}$	$Q_{\varphi\theta}$	$Q_{\varphi\varphi}$
q_{xx}	q_{xy}	$q_{x\theta}$	$q_{x\varphi}$
q_{yx}	q_{yy}	$q_{y\theta}$	$q_{y\varphi}$
$q_{\theta x}$	$q_{\theta y}$	$q_{\theta\theta}$	$q_{\theta\varphi}$
$q_{\varphi x}$	$q_{\varphi y}$	$q_{\varphi\theta}$	$q_{\varphi\varphi}$

These are the gradients of the timevarying magnetic forces and moments:

$Q_{xx}, Q_{xy}, Q_{yx}, Q_{yy}, q_{xx}, q_{xy}, q_{yx}, q_{yy}$ in lbs/inch

$Q_{x\theta}, Q_{x\phi}, Q_{y\theta}, Q_{y\phi}, q_{x\theta}, q_{x\phi}, q_{y\theta}, q_{y\phi}$ in lbs/radian

$Q_{\theta x}, Q_{\theta y}, Q_{\phi x}, Q_{\phi y}, q_{\theta x}, q_{\theta y}, q_{\phi x}, q_{\phi y}$ in lbs-inch/inch

$Q_{\theta\theta}, Q_{\theta\phi}, Q_{\phi\theta}, Q_{\phi\phi}, q_{\theta\theta}, q_{\theta\phi}, q_{\phi\theta}, q_{\phi\phi}$ in lbs-inch/radian

b. If KC=0, give 4 cards with 2 values per card

$KA > 0$

$Q_{xx} \quad Q_{xy}$

$Q_{yx} \quad Q_{yy}$

$q_{xx} \quad q_{xy}$

$q_{yx} \quad q_{yy}$

$KA < 0$

$Q_{ex} \quad Q_{ey}$

$Q_{\phi x} \quad Q_{\phi y}$

$q_{ex} \quad q_{ey}$

$q_{\phi x} \quad q_{\phi y}$

c. If KC=1, give 4 cards with 2 values per card

$KA > 0$

$Q_{x\theta} \quad Q_{x\phi}$

$Q_{y\theta} \quad Q_{y\phi}$

$q_{x\theta} \quad q_{x\phi}$

$q_{y\theta} \quad q_{y\phi}$

$KA < 0$

$Q_{\theta\theta} \quad Q_{\theta\phi}$

$Q_{\phi\theta} \quad Q_{\phi\phi}$

$q_{\theta\theta} \quad q_{\theta\phi}$

$q_{\phi\theta} \quad q_{\phi\phi}$

Potor Eccentricity and Misalignment Coordinates (IP4E14.6)

Give one card with either 4 or 2 values on it:

a. If KC=-1:

1. x_0 , eccentricity of rotor in alternator centerplane, x-direction, inch
2. y_0 , eccentricity of rotor in alternator centerplane, y-direction, inch
3. θ_0 , misalignment angle in x-plane, radians or inches/inch
4. ϕ_0 , misalignment angle in y-plane, radians or inches/inch

b. If KC=0:

1. x_0 , inches
2. y_0 , inches

c. If KC=1:

1. θ_0 , inches/inch
2. ϕ_0 , inches/inch

Bearing Data, Fixed Geometry (8E3.2)

Applies when NPD=0 (card 2, item 6). If the lubricant is incompressible (INC=0; card 2, item 7), give one card per bearing. If the lubricant is compressible (INC=1), give (NH+1)-cards per bearing (NH is item 8, card 2). Each card gives a set of 8 bearing coefficients:

1. Spring coefficient K_{xx} , lbs/inch
2. Damping ωB_{xx} , lbs/inch
3. Spring coefficient K_{xy} , lbs/inch
4. Damping ωB_{xy} , lbs/inch
5. Spring coefficient K_{yx} , lbs/inch
6. Damping ωB_{yx} , lbs/inch
7. Spring coefficient K_{yy} , lbs/inch
8. Damping ωB_{yy} , lbs/inch

Bearing Data, Tilting Pad Bearing

Applies when NPD $\neq 0$ (card 2, item 6). Define the number NPDI by:

$$\text{if } NPD \geq 1 \text{ (load between pads): } \begin{cases} NPD \text{ even, then: } NPDI = 1/2 \cdot NPD \\ NPD \text{ odd, then: } NPDI = 1/2 \cdot (NPD + 1) \end{cases}$$

$$\text{if } NPD \leq -1 \text{ (load on pad): } \begin{cases} |NPDI| \text{ even, then: } NPDI = 1/2 \cdot |NPDI| + 1 \\ |NPDI| \text{ odd, then: } NPDI = 1/2 \cdot (|NPDI| + 1) \end{cases}$$

NPDI is the number of pads for which input is required per bearing. If the lubricant is incompressible (INC=0; card 2, item 7), give two cards per pad. If the lubricant is compressible (INC=1), give (NH+2)-cards per pad. In either case the first card is:

(1P5E14.6)

1. Pitch mass moment of inertia divided by the square of the journal radius, lbs
2. Pad mass, lbs
3. Radial stiffness of pivot support, lbs/inch
4. Angle from bearing load line to pivot point, degrees.

Then follow 1 card if INC=0, or (NH+1)-cards if INC=1, with the 8 dynamic coefficients for the pad:

Card: (8E9.2)

1. Spring coefficient K_{xx} , lbs/inch
2. Damping ωB_{xx} , lbs/inch
3. Spring coefficient K_{xy} , lbs/inch
4. Damping ωB_{xy} , lbs/inch
5. Spring coefficient K_{yx} , lbs/inch
6. Damping ωB_{yx} , lbs/inch
7. Spring coefficient K_{yy} , lbs/inch
8. Damping ωB_{yy} , lbs/inch

These (NH+2)-cards must be repeated NPD1 times per bearing, and there must be one complete set for each bearing (there are NB bearings).

- EPN SOURCE STATEMENT - IUN(S) -

514-1967 J. LUND MECHANICAL TECHNOLOGY INC.
 PM354=RESPONSE OF ROTOR WITH MAGNETIC FORCES
 COMMON/A/ BKXX(1C,1),BCXX(10,11),BKXY(10,11),BCXY(10,11),
 1BKYX(10,11),BCVX(10,11),BKYY(10,11),BCYY(10,11),PMINC(0,5),
 2FADM(10,5),PADK(10,5),PANG(10,5),DEVN(4,4),WCC(4,4),CPR(4,8),
 3CRE(4,8),AKR(4,8),AKE(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
 4WCT(4,4),WQ(4,4),WSU(4,4),UR(4,4),UE(4,4),EMR(4,4),EME(4,4),
 5XR(4,1),XE(4,11)
 COMMON/B/ PKXX(10,5,11),PCXX(10,5,11),PKXY(10,5,11),PCXY(10,5,11),
 1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
 2GR(2,8,50),GE(2,8,50),SR(4,4,1),SE(4,4,1),XCS(4,50,1),
 3XSS(4,50,1),YCS(4,50,11),YSS(4,50,11)
 COMMON/C/ RM(30),RIP(30),RIT(30),RS(30),RW(30),RD(30),RL(30),
 1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
 2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),B_(30),
 3B2(30),B3(30),B4(30),B5(30),B6(30),B7(30),B8(30),B9(30),B10(30),
 4PMX(10),PDX(10),PMY(10),PKY(10),PDY(10),SXX(10),DXX(10),
 5SXY(10),DRY(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
 6XZ(4),XZ(4)
 COMMON/D/A1,A2,A3,A4,A5,A6,A7,A8,NF,FRQ,KC,SCFL,NDIA,KQ2,NS,KB,
 * IK,NH2,MN,AMLM,SPU,SFRI,SFR,WTST,WTIN,WTF, NH1
 COMMON/E/KUL,KQ3,C_,C2,NB,KA,NRP,NPU,INC,NH,NSP,INP,YM,DNST,SHM,
 *SPST,SPFN,SPIN,SCF,QZ,QZP,K_,K2, WTN
 COMMON/F/BMXC,BMXS,BMYC,BMYS,VXG,VXS,VYC,VYS,XC,XS,YC,YS,DXC,DXS,
 *DYS,C3,C4, NS_, NPU, NPD2, UYC, NSPI
 WRITE(5,99)
 190 READ(5,10_1)
 READ(5,10_1) NS, NB, KA, KC, NRP, NPD, INC, NH, NSP, NDIA, INP
 READ(5,10_1) YM, DNST, SHM
 WRITE(6,10_1)
 WRITE(6,10_1)
 WRITE(6,10_1) NS, NB, KA, KC, NRP, NPU, INC, NH, NSP, NDIA, INP
 WRITE(6,10_1)
 WRITE(6,10_1) YM, DNST, SHM
 DNST=DNST/386.069
 NS1=NS_
 NH1=NH_+
 IF(KC) 196,195,199
 195 KQ1=4
 KQ2=2
 KQ3=6
 GO TO .97
 196 KQ1=8
 KQ2=4
 KQ3=8
 197 IF(KA) 198,199,199
 198 KB=-KA
 GO TO 200
 199 KB=KA
 200 WRITE(6,110)
 WRITE(6,110)
 DO 203 J=1,NS
 READ(5,10_1) RM(J), RIP(J), RIT(J), RL(J), RS(J), RW(J), RD(J)
 WRITE(6,10_1) J,RM(J),RIP(J),RIT(J),RL(J),RS(J),RW(J),RD(J)
 RM(J)=RM(J)/386.069

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TWO - EFM SOURCE STATEMENT - FFM(FI) -

RIP(J)=RIP(J)/386.069	45
RIT(J)=RIT(J)/386.069	46
C1=0.049087183*YM*(RS(J)*+4-RD(J)**4)	47
RW(J)=0.785398 6*D-STM(RW(J)**2-RD(J)**2)	48
C2=1.5707963*SHM*(HS(J)**2-HD(J)**2)	49
RS(J)=C	50
IF(C2) 202,202,30.	51
201 RD(J)=C1/C2	52
GO TO 203	53
202 RD(J)=0.0	54
203 CONTINUE	55
READ (5,101) (LB(J), J=1,NB)	56
WRITE(5,104)	57 91
WRITE(6,101)(LB(J),J=1,NB)	58 92
IF(NRP) 210,210,204	59
204 WRITE(6,111)	60 100
WRITE(6,112)	61 101
DO 205 J=1,NB	62
READ (5,105) PMX(J), PKX(J), PDX(J), PMY(J), PKY(J), PDY(J)	63
WRITE(6,107) LB(J),PMX(J),PKX(J),PDX(J),PMY(J),PKY(J),PDY(J)	64 111
PMX(J)=PMX(J)/386.069	65
205 PMY(J)=PMY(J)/386.069	66
210 IF(NPD) 214,217,211	67
211 IF((NPD/2)*2-NPD) 213,212,212	68
212 NPD1=NPD/2	69
NPD2=-2	70
GO TO 213	71
213 NPD1=(NPD+1)/2	72
NPD2=-1	73
GO TO 217	74
214 NPD1=-NPD	75
IF((NPD1/2)*2-NPD.) 216,215,215	76
215 NPD1=NPD1/c+1	77
NPD2=0	78
GO TO 217	79
216 NPD1=(NPD1+1)/2	80
NPD2=1	81
217 NSP1=1	82
READ (5,102) WTST, WTFN, WTIN, AMLM	145
230 READ (5,102) SPST, SPFN, SPIN, SFR, SCF	146
RFAD (5,102) QZ, QZP	147
WRITE(6,113)	148
WRITE(6,114)	149
WRITE(6,102) QZ,QZP	150
WRITE(6,115)	151
DO 218 I=1,KQ2	
READ (5,138) (WQ(I,J), J=1,KQ2)	154
WRITE(6,138) (WQ(I,J),J=1,KQ2)	159
218 CONTINUE	
WRITE(6,126)	166
DO 219 I=1,KQ2	
READ (5,138) (WSQ(I,J),J=1,KQ2)	169
WRITE(6,138)(WSQ(I,J),J=1,KQ2)	174
219 CONTINUE	
READ (5,138) (XZ(I), I=1,KQ2)	181
WRITE(6,129)	188

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THC - EFN SOURCE STATEMENT - IFN(S) -

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      WRITE(6,103)(X2(I),I=1,NQ2)          189
      SP2=0.347.97e+5t4
      WRITE(6,104)
      IF(INC) 242,244,245
244. K2=,
      GO TO 243
242 K2=NH1
243 DO 255 J=1,NB
      WRITE(6,107)LB(J)
      IF(NPD) 255,244,255
244 WRITE(6,110)
      DO 245 I=1,K1
      K2=I-1
      READ (5,105) BKXX(J,I), BCXX(J,I), BKXY(J,I),BCXY(J,I),BKYX(J,I),
      BCYX(J,I), BKYY(J,I), BCYY(J,I)
      WRITE(6,107)K2,BKXX(J,I),BCXX(J,I),BKXY(J,I),BCXY(J,I),BKYX(J,I),
      BCYX(J,I),BKYY(J,I),BCYY(J,I)
245 CONTINUE
      IF(INC) 255,246,255
246 DO 247 I=2,NH1
      BKXX(J,I)=BKXX(J,1)
      BCXX(J,I)=BCXX(J,1)
      BKXY(J,I)=BKXY(J,1)
      BCXY(J,I)=BCXY(J,1)
      BKYX(J,I)=BKYX(J,1)
      BCYX(J,I)=BCYX(J,1)
      BKYY(J,I)=BKYY(J,1)
247 BCYY(J,I)=BCYY(J,1)
      GO TO 255
250 DO 254 K=1,NPD
      WRITE(6,119)K
      WRITE(6,120)
      READ (5,102) PMIN(J,K),PADM(J,K), PADK(J,K), PANG(J,K)
      WRITE(6,102)PMIN(J,K),PADM(J,K),PADK(J,K),PANG(J,K)
      PMIN(J,K)=PMIN(J,K)/386.069
      PADM(J,K)=PADM(J,K)/386.069
      PANG(J,K)=0.017453293*PANG(J,K)
      WRITE(6,118)
      DO 251 I=1,K1
      K2=I-1
      READ (5,106) PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),
      PKYX(J,K,I), PCYX(J,K,I), PKYY(J,K,I),PCYY(J,K,I)
      WRITE(6,107)K2,PKXX(J,K,I),PCXX(J,K,I),PKXY(J,K,I),PCXY(J,K,I),
      1PKYX(J,K,I),PCYX(J,K,I),PKYY(J,K,I),PCYY(J,K,I)
251 CONTINUE
      IF(INC) 254,252,254
252 DO 253 I=2,NH1
      PKXX(J,K,I)=PKXX(J,K,1)
      PCXX(J,K,I)=PCXX(J,K,1)
      PKXY(J,K,I)=PKXY(J,K,1)
      PCXY(J,K,I)=PCXY(J,K,1)
      PKYX(J,K,I)=PKYX(J,K,1)
      PCYX(J,K,I)=PCYX(J,K,1)
      PKYY(J,K,I)=PKYY(J,K,1)
253 PCYY(J,K,I)=PCYY(J,K,1)
254 CONTINUE

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TWO	- EFN SOURCE STATEMENT -	IFN(S)
255	CONTINUE	150
	SPD=SPST	151
260	WRITE(6,12.)SPD	152
	SCFI=0.10471976*SPD	323
	SCFI=SCF*SCF.*SCFI	153
	C1=0.5/SCF.	154
DO 261	I=1,KQ2	
DO 261	J=1,KQ2	
	SR(I,J,NH)=0.0	
	SE(I,J,NH)=0.0	
	WB(I,J)=C1*WQ(I,J)	
261	WC(I,J)=-C.*WSQ(I,J)	
	CALL AA2 (\$260,\$230,\$490)	339
99	FORMAT (1H.)	
100	FORMAT(72H	
)	
101	FORMAT(11I9)	740
102	FORMAT(1P5E14.6)	741
103	FORMAT(108HOSTATIONS BEARINGS MAGN.ST F+M/F/M RIG.PED NO. 1PADS COMPRESS HARMONICS NO.SPEEDS DIAGNOS INPUT)	742
104	FORMAT(16,_0I10)	743
105	FORMAT(42H0 YOUNGS MOD. DENSITY (SHAPE FACT)*G)	746
106	FORMAT(8E9.2)	747
107	FORMAT(15,_1PE16.6,_1P7E14.6)	748
108	FORMAT(104H STATION MASS,LBS POLAR MOM.IN. TRANSV.MOM.IN LENGTH OUT,DIA(STIFF) OUT,DIA(MASS) INNER DIA.)	L 749
109	FORMAT(17H0BEARING STATIONS)	750
110	FORMAT(11H0ROTOR DATA)	751
111	FORMAT(14H0PEDESTAL DATA)	752
112	FORMAT(89H STATION MASS-X,LBS STIFFNESS-X DAMPING-X MASS 1-Y,LBS STIFFNESS-Y DAMPING-Y)	753
113	FORMAT(20H1MAGNETIC FORCE DATA)	754
114	FORMAT(27H) Q(0),FORCE Q(0),MOMENT	755
115	FORMAT(28HOMATRIX OF COSINE COMPONENTS)	
116	FORMAT(//13H0BEARING DATA)	756
117	FORMAT(19H0BEARING AT STATION,I3)	757
118	FORMAT(9H0HARMONIC4X3HKXX.0X5HW*BXX.0X3HKXY10X5HW=BXY.0X3HKYY10X5H 1W=BXY10X3HKYY10X5HW=BYY)	760
119	FORMAT(8H0PAD NO.,I3)	761
120	FORMAT(55H PITCH MUM,IN. PAD MASS PIVOT STIFFN. PIVOT ANGLE)	762
121	FORMAT(13H,ROTOR SPEED=,1PE13.6,4H RPM)	763
122	FORMAT(//13H0HARMONIC NO.,I3,_1H,FREQUENCY=,1PE13.6,8H RAD/SEC)	764
123	FORMAT(21H0BEARING COEFFICIENTS)	765
124	FORMAT(8H STATION5X3HKXX9X7HFRQ=BXX9X3HKXY9X7HFRQ=BXY9X3HKYY9X7HFR .Q=8YX9X3HKYY9X7HFRQ=BYY)	766
125	FORMAT(31H0ROTOR-BEARING IMPEDANCE MATRIX)	767
126	FORMAT(26H)MATRIX OF SINE COMPONENTS)	768
127	FORMAT(10HREAL PART)	769
128	FORMAT(15H)IMAGINARY PART)	770
129	FORMAT(4LHCORDINATES OF STATIC ROTOR ECCENTRICITY)	771
130	FORMAT(34HREAL PART OF A-MATRIX IS SINGULAR)	
131	FORMAT(10H HARMONIC=,I3,12H FREQUENCY=,1PE13.6)	
132	FORMAT(18H,ROTOR STATION NO.,I2)	
133	FORMAT(25HFINAL MATRIX IS SINGULAR)	
134	FORMAT(49H)AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES)	
135	FORMAT(12H HARMONIC FREQUE ICY7X4HX(C)10X4HX(S)10X4HY(C)10X4HY(S))	

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TWO - EFM SOURCE STATEMENT - IFN(S) -

```
136 FORMAT(//.2H0WHIRL ORBIT)
137 FORMAT(.6H SHAFT RUTAT,DEG5X1HX13X1HY9X9HAMPLITUDE4X12'ANGLE X-AMP
1L)
138 FORMAT(1P4E14.6)
139 FORMAT(2.6H HARMONIC FREQUENCY5X4HX(C)8X4HX(S)8X4HY(C)8X4HY(S)6X8
1HX/DZ(C)+X8HDY/DZ(S)+X8HDY/DZ(C)+X8HDY/DZ(S))
140 FORMAT(42HREAL PART OF IMPEDANCE MATRIX IS SINGULAR)
141 FORMAT(15.1PE15.4,1P8E12.4)
142 FORMAT(37HREAL PART OF S-MATRIX IS SINGULAR,K=,13)
143 FORMAT(25HINVERSE IMPEDANCE MATRIX)
144 FORMAT(54HINFLUENCE COEFFICIENT MATRIX E (X(J)=E+F) FOR STATION,I
3)
145 FORMAT(46HINFLUENCE MATRIX A (X(J)=A*X(KB)) FOR STATION,13)
146 FORMAT(35HINVERSION MATRIX FOR A IS SINGULAR)
147 FORMAT(35HINVERSION MATRIX FOR E IS SINGULAR)
148 FORMAT(38HINVERSION MATRIX FOR S IS SINGULAR,K=13)
STOP
END
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SAL - EFN SOURCE STATEMENT - IFN(S) -

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SUBROUTINE AA2 (*,*,*)
COMMON/A/ BKXX(10,11),BCXX(10,11),BCKY(10,11),BCXY(10,11),
1BKX(10,11),BCYX(10,11),BKYY(10,11),BCYY(10,11),PMINI(0,5),
2PAOM(10,5),PAUK(10,5),PANG(10,5),DEVN(4,4),WCC(4,1),CMR(4,8),
3CME(4,8),AMR(4,8),AME(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
4WC(4,4),WO(4,4),WSQ(4,4),UR(4,4),UE(4,4),EMR(4,1),EME(4,4),
5XR(4,11),XE(4,11)
COMMON/B/ PKXX(10,5,1.),PCXX(10,5,1.),PKXY(10,5,11),PCXY(10,5,11),
1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
2GR(2,8,50),GE(2,8,50),SR(4,4,1),SE(4,4,1),XCS(4,50,1),
3SS(4,50,11),YCS(4,50,11),YSS(4,50,11)
COMMON/C/ RM(30),RIP(30),RF(30),RS(30),RW(30),RD(30),RL(30),
1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),BL(30),
3B2(30),B3(30),B4(30),B5(30),B6(30),B7(30),B8(30),B9(30),B10(30),
4PMX(10),PKX(10),PDX(10),PMY(10),PKY(10),PDY(10),SXX(10),DX(10),
5SX(10),DX(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
6Z(4),XZ(4)
COMMON/D/A,A2,A3,A4,A5,A6,A7,AB,NF,FRQ,KC,SCF1,NDIA,KQ2,NS,KB,
* IK,NH2,HN,AHLM,SPD,SFR1,SFR,WTST,WTIN,WTF, NH1
COMMON/E/KQ1,KQ3,C1,C2,NR,KA,NRP,NPD,INC,NH,NSP,INP,YM,DNST,SHM,
*SPST,SPFN,SPIN,SCF,QZ,QZP,K1,K2, WTN
COMMON/F/BMXC,BMXS,BMYC,BMYS,VXC,VXS,VYC,VYS,XG,XS,YC,YS,DXC,DXS,
*DYS,C3,C4, NS1, NPD1, NPD2, DYC, NSP1
DO 530 IK=1,NH1
IH=NH1+1-IK
IH=NH1
NF=IH-1
HN=NF
FRQ=HN*SPD*SFR1
FQ2=FRQ*FRQ
HN1=HN*SFR
IF(NCIA) 401,402,404
401 WRITE(6,122)NF,FRQ
        WRITE(6,123)
        WRITE(6,124)
402 DO 425 J=1,NB
        IF(NPD)404,403,404
403 D1=BKXX(J,IH)
        D2=BCXX(J,IH)*HN1
        D3=BKXY(J,IH)
        D4=BCXY(J,IH)*HN1
        D5=BKYY(J,IH)
        D6=BCYY(J,IH)*HN1
        D7=BKYY(J,IH)
        D8=BCYY(J,IH)*HN1
        GO TO 406
404 D1=0.0
        D2=0.0
        D3=0.0
        D4=0.0
        D5=0.0
        D6=0.0
        D7=0.0
        D8=0.0
        DO 415 I=1,NPD1
        157
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SAL : FN SOURCE STATEMENT - IFN(S) -

C1=FQ2*PMIN(J,I)	185
A1=PKXX(J,I,IH)	186
A2=PCXX(J,I,IH)*HN.	187
A3=PKXY(J,I,IH)	188
A4=PCXY(J,I,IH)*HN.	189
A5=PKYY(J,I,IH)	190
A6=PCYY(J,I,IH)*HN.	191
A7=PKYY(J,I,IH)	192
A8=PCYY(J,I,IH)*HN.	193
C2=AT-C.	194
C3=PAUK(J,I)-FQ2*PADM(J,I)	195
C4=A.+C?	196
C5=C4+C2-A1+A8-A3+A5+A4+A6	197
C6=C4+A8+C2-A2-A3+A6-A5+A4	198
C7=C5+C6+C0	199
C_.R=C3*(C_+C2+C6*A8)/C7	200
C11E=C3*(C_+A8-C5+C2)/C7	201
C_.2R=C_.(C5+A3+C6*A+1)/C7	202
C12E=C1*(C5+A4-C6*A5)/C7	203
C21R=-C3*(C5*A5+C6*A6)/C7	204
C21E=C3*(C6*A5-C5*A6)/C7	205
C22R=-C_.*(C5*C4+C6*A2)/C7	206
C22E=C_.*(C4+C6-C5*A1)/C7	207
DKXX=C_.R*A1-C11E*A2+C21R*A3-C21E*A4	208
DCXX=C_.R*A1+C11E*A1+C21R*A4+C21E*A3	209
DKXY=C_.2R*A1-C_.2E*A2+C22R*A3-C22E*A4	210
DCXY=C_.2R*A1+C_.2E*A1+C22R*A4+C22E*A3	211
DKYX=C_.R*A5-C11E*A5+C21R*A3+C21E*A7	212
DCYX=C_.R*A6+C11E*A5+C21R*A3+C21E*A7	213
DKYY=C_.2R*A5-C12E*A6+C22R*A7-C22E*A8	214
DCYY=C_.2R*A6+C12E*A6+C22R*A8+C22E*A7	215
C3=PANG(J,I)	216
C1=CCS(C3)	217
C2=SIN(C3)	218
C4=C1*C	41
C5=C2*C.	219
A1=DKXX*C4+DKYY*C5	220
A2=DCXX*C4+DCYY*C5	221
A3=DKXY*C4+DKYY*C5	222
A4=DCXY*C4+DCYY*C5	223
A5=DKYX*C4+DKXX*C5	224
A6=DCYX*C4+DCXX*C5	225
A7=DKYY*C4+DKXX*C5	226
A8=DCYY*C4+DCXX*C5	227
IF(NPD2+1)=400,405,400	228
405 IF(I-1)=400,406,407	229
406 IF(NPD2)=409,409,400	230
407 IF(I-NPD1)=410,418,408	231
408 IF(NPD2)=410,409,409	232
409 D1=D1+A.	233
D2=D2+A2	234
D3=D3+A3	235
D4=D4+A4	236
D5=D5+A5	237
D6=D6+A6	238
D7=D7+A7	239
	240

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SAL - EFM SOURCE STATEMENT - IFN(S) -

D8=D8+A8	241
GO TO 415	242
410 D1=D1+A1+A1	243
D2=D2+A2+A2	244
D3=D3+A3+A3	245
D4=D4+A4+A4	246
D5=D5+A5+A5	247
D6=D6+A6+A6	248
D7=D7+A7+A7	249
D8=D8+A8+A8	250
415 CONTINUE	251
416 IF(NDIA) 417,420,417	252
417 WRITE(6,107)LB(J),D1,D2,D3,D4,D5,D6,D7,D8	253 58
420 IF(NRP) 421,422	254
421 SXX(J)=D1	255
DXX(J)=D2	256
SXY(J)=D3	257
DXY(J)=D4	258
SYX(J)=D5	259
DYX(J)=D6	260
SYY(J)=D7	261
DYY(J)=D8	262
GO TO 425	263
422 C1=PKX(J)-FQ2*PMX(J)	264
C2=PKY(J)-FQ2*PMY(J)	265
C3=FRQ*PDY(J)	266
C4=FRQ*PDY(J)	267
C5=D1+C1	268
C6=D7+C2	269
C7=D2+C3	270
C8=D8+C4	271
A1=C5*C6-C7*C8-D3*D5+D4*D6	272
A2=C5*C8+C6*C7-D3*D6-D4*D5	273
C9=A1+A1+A2+A2	274
A3=C1*C6-C3*C8	275
A4=C1*C8+C3*C6	276
C11R=(A3*A1+A4*A2)/C9	277
C11E=(A4*A1-A3*A2)/C9	278
A3=C2*D3-C4*D4	279
A4=C2*D4+C4*D3	280
C12R=(A3*A1+A4*A2)/C9	281
C12E=(A4*A1-A3*A2)/C9	282
A3=C1*D5-C3*D6	283
A4=C1*D6+C3*D5	284
C21R=(A3*A1+A4*A2)/C9	285
C21E=(A4*A1-A3*A2)/C9	286
A3=C2*C5-C4*C7	287
A4=C2*C7+C4*C5	288
C22R=(A3*A1+A4*A2)/C9	289
C22E=(A4*A1-A3*A2)/C9	290
SXX(J)=C11R*D1-C11E*D2+C21R*D3-C21E*D4	291
DXX(J)=C11R*D2+C11E*D1+C21R*D4+C21E*D3	292
SXY(J)=C12R*D1-C12E*D2+C22R*D3-C22E*D4	293
DXY(J)=C12R*D2+C12E*D1+C22R*D4+C22E*D3	294
SYX(J)=C11R*D5-C11E*D6+C21R*D7-C21E*D8	295
DYX(J)=C11R*D6+C11E*D5+C21R*D4+C21E*D7	296

SAL - IFN SOURCE STATEMENT - IFN(S) -

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SYX(J)=C.2R*D5-C.2S*D6+C?2R*D7-C22E*D8	297
DYY(J)=C.2R*D5+C.2S*D6+C22R*D8+C22E*D7	298
425 CONTINUE	299
DO 442 J=1,NS.	300
C1=RS(J)	301
C2=FC2*HW(J)	302
C3=LU(J)	303
C4=C2/C.	304
C5=SORT(C4)	305
C6=SURT(C5)	306
C7=RL(J)	307
IF(C6>C7-.03) 441,442	308
441 C8=C2*C7	309
B1(J)=.C	310
B2(J)=.C	311
B3(J)=C7	312
B6(J)=C7/C.	313
B4(J)=B6(J)/2.0*C7	314
B7(J)=B6(J)/3.0*C7-C3*C7/C1*2.0	315
B5(J)=C2*B7(J)	316
B8(J)=C8	317
B9(J)=C8/2.0*C7	318
B10(J)=B9(J)/3.0*C7	319
GO TO 449	320
442 C8=C3+C3*C4	321
C9=.3*C4	322
IF(C8-0.00<.) 443,443,444	323
443 C8=.0+J.5*C8	324
GO TO 445	325
444 C8=SQRT(1.0+C8)	326
445 A5=C5*(C8-C9)	327
A6=C5*(C8+C9)	328
A9=A5+A6	329
A3=SORT(A5)	330
A4=SORT(A6)	331
A7=A3*A5	119
A8=A4*A6	332
A1=A3*C7	333
A2=A4*C7	334
TWOP1 = 2.*3.1415926	335
D1= COS(AMOD(A2,TWOP1))/A9	123
D2= SIN(AMOD(A2,TWOP1))/A9	124
D5=EXP(A1)	125
D4=1.0/D5	338
D3=0.5*(D5+D4)/A9	339
D4=0.5*(D5-D4)/A9	340
B1(J)=A5*D2+A6*D4	341
B2(J)=A6*D3+A5*D1	342
B3(J)=A3*D4+A4*D2	343
B8(J)=C2*B1(J)	344
B4(J)=(D3-D1)/C1	345
B9(J)=C2*(D3-D1)	346
B5(J)=C5*(A4*D4-A3*D2)	347
B10(J)=C1*B5(J)	348
C8=C1*C5	349
B6(J)=(A8*D4+A7*D2)/C8	350
	351

SAL - EFN SOURCE STATEMENT - IFN(S) -

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	87(J)=(A7*D4-A8*D2)/C4	
449	CONTINUE	352
	DO 455 J=1,NS	353
	C1=FQ2*RM(J)	354
	DVXA(J)=C1	355
	DVYA(J)=C1	356
	DVXB(J)=0.0	357
	DVXC(J)=0.0	358
	DVXD(J)=0.0	359
	DVYB(J)=0.0	360
	DVYC(J)=0.0	361
	DVYD(J)=0.0	362
	DMXA(J)=FQ2*RIT(J)	363
	DVUX(J)=0.0	364
	DVUY(J)=0.0	365
	DMUX(J)=0.0	366
455	DMUY(J)=0.0	367
	DO 456 J=1,NB	368
	K1=LE(J)	369
	DVXA(K1)=DVXA(K1)-SXX(J)	370
	DVXB(K1)=DX(X(J)	371
	DVXC(K1)=SXY(J)	372
	DVXD(K1)=DXY(J)	373
	DVYA(K1)=DVYA(K1)-SYY(J)	374
	DVYB(K1)=DYY(J)	375
	DVYC(K1)=SYX(J)	376
456	DVYD(K1)=DXY(J)	377
	DVXA(KB)=DVXA(KB)+QZ	378
	DVYA(KB)=DVYA(KB)+QZ	379
	DMXA(KB)=DMXA(KB)+QZP	380
	CALL BB2	381
	DO 486 J=1,4	189
	DO 486 I=1,4	472
	WR(I,J)=AMR(I,J)	473
486	WE(I,J)=AME(I,J)	474
	CALL MATINV(WR,4,WCC,0,DVN,1D)	475
	GO TO(481,460),ID	202
460	WRITE(6,_3J)	204
	WRITE(_,_3J,INF,FRQ	205
	GO TO 530	
481	IF(NF) 457,457,482	
457	DO 458 I=1,4	
	DO 458 J=1,4	
	AME(I,J)=0.0	
	UR(I,J)=WR(I,J)	
458	UE(I,J)=0.0	
	GO TO 459	
482	CALL MATINV(WE,4,WCC,0,DVN,1D)	223
	GO TO(453,457),ID	
483	DO 488 I=1,4	
	DO 488 J=1,4	
	C1=0.0	
	DO 487 K=1,4	
487	C1=C1+UR(I,K)*AME(K,J)+WE(I,K)*AMR(K,J)	479
488	WA(I,J)=C1	480
	CALL MATINV(WA,4,WCC,0,DVN,1D)	481
		482
		483
		242

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GO TO (705,702),10
702 WRITE(6,4)
WRITE(6,13)INF,FRG
GO TO 3-3
703 DO 490 I=1,4
DO 490 J=1,4
C1=0.0
C2=0.0
DO 489 K=1,4
C1=C1+WA(I,K)*WC(K,J)
492 C2=C2-WA(I,K)*WR(K,J)
UR(I,J)=C1
493 UE(I,J)=C2
499 DO 494 I=1,4
DO 492 J=1,KQ3
C1=0.0
C2=0.0
DO 494 K=1,4
C1=L.-UR(I,K)*AMR(K,J)+UE(I,K)*AME(K,J)
495 C2=C1-UR(I,K)*AME(K,J)-UF(I,K)*AMR(K,J)
WR(I,J-4)=C1
492 WE(I,J-4)=C2
DO 494 I=1,4
DO 494 J=1,KQ2
C1=CMR(I,J+4)
C2=CME(I,J+4)
DO 493 K=1,4
C1=C1+CMR(I,K)*WR(K,J)-CME(I,K)*WE(K,J)
493 C2=C2+CMR(I,K)*WE(K,J)+CME(I,K)*WR(K,J)
UR(I,J)=C1
494 UE(I,J)=C2
IF(NDIA) 704,705,705
704 WRITE(6,143)
WRITE(6,271)
WRITE(6,36)((UR(I,J),J=1,KQ2),I=1,4)
WRITE(6,26)
WRITE(6,38)((UE(I,J),J=1,KQ2),I=1,4)
705 IF(KC) 61,610,61
610 DO 497 I=1,2
DO 497 J=1,2
IF(KC) 61,495,493
495 EMR(I,J)=UR(I,J)
EME(I,J)=UE(I,J)
GO TO 497
496 EMR(I,J)=UR(I+2,J)
EME(I,J)=UE(I+2,J)
497 CONTINUE
C1=EMR(1,1)+EMR(2,1)-EMR(1,2)*EMR(2,1)-EME(1,1)*EME(2,2)+  

-EME(1,2)*EME(2,1)
C2=EMR(1,1)*EME(2,1)+EMR(2,1)*EME(1,1)-EMR(1,2)*EME(2,1)-  

-EMR(2,1)*EME(1,2)
C3=(C1+C1+C2+C2)*SCF
C1=C1/C3
C2=C2/C3
UR(1,1) =C1+EMR(1,2)+C2*EME(2,2)
UR(1,2) =-C1+EMR(1,2)-C2*EME(1,2)

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SAL - EPN SOURCE STATEMENT - [FN(S)] -

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UR(2,1) =C1*EMR(1,1)+C2*EMR(2,1)
UR(2,2) = C1*EMR(1,1)+C2*EMR(1,2)
UE(1,1) = C1*EME(2,1)-C2*EMR(2,2)
UE(1,2) =-C1*EME(1,2)+C2*EMR(1,2)
UE(2,1) =-C1*EME(2,1)+C2*EMR(2,1)
UE(2,2) = C1*EME(1,1)-C2*EMR(1,1)
GO TO 623
611 DO 612 I=1,4
DO 612 J=-4
EMR(I,J)=UR(I,J)
612 EME(I,J)=UE(I,J)
CALL MATINV(EMR,4,WCC,C,DVN,1D) 365
GO TO(614,613),ID
613 WRITE(6,140) 367
WRITE(6,131) NF,FRQ 368
GO TC 530
614 IF(NF) 615,615,617
615 DO 616 I=1,4
DO 616 J=-4
UR(I,J)=EMR(I,J)/SCFI
616 UE(I,J)=0.0
GO TO 623
617 CALL MATINV(EME,4,WCC,C,DVN,1D) 385
GO TO(618,615),ID
618 DO 620 I=1,4
DO 620 J=-4
C1=0.0
DO 619 K=1,4
619 C1=C1+EMR(I,K)*UE(K,J)+EME(I,K)*UR(K,J)
620 WA(I,J)=C1
CALL MATINV(WA,4,WCC,C,DVN,1D) 404
GO TO (707,706),ID
706 WRITE(6,-47) 406
WRITE(6,-31)NF,FRQ 407
GO TC 530
707 DO 622 I=-4
DO 622 J=1,4
C1=0.0
C2=0.0
DO 621 K=1,4
C1=C1+WA(I,K)*EME(K,J)
621 C2=C2-WA(I,K)*EMR(K,J)
UR(I,J)=C1/SCFI
622 UE(I,J)=C2/SCFI
623 IF(NDIA) 498,499,498
498 WRITE(6,-23)
WRITE(6,-21)
WRITE(6,136)((UR(I,J),J=1,4),I=-4)
WRITE(6,-20)
WRITE(6,-30)((UE(I,J),J=1,4),I=-4) 429
430
431
439
440
499 DO 504 J=-NS
DO 501 I=1,2
DO 501 K=1,KQ2
C1=GR(I,K+1,J)
C2=GE(I,K+1,J)
DO 500 L=1,4

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SAL - EPI SOURCE STATEMENT - IFN(S) -

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C = C + G(I,L,J)*W(L,K) + G(I,L,J)*W(L,K)
501 C = C + G(I,L,J)*W(L,K) + G(I,L,J)*W(L,K)
CM(I,K) = C
502 CM(I,K) = C
DO 503 I = 1, KQ2
C = J,I
C1 = U(I,J)
C2 = U(I,J)
C3 = U(I,J)
C4 = U(I,J)
DO 502 K = 1, KQ2
C1 = C + CMR(I,K)*UM(K,I) - CME(I,K)*UL(K,I)
C2 = C + CMR(I,K)*UL(K,I) + CME(I,K)*UR(K,I)
C3 = C + CMR(I,K)*UR(K,I) - CME(I,K)*UL(K,I)
503 C4 = C + CMR(I,K)*UL(K,I) + CME(I,K)*UR(K,I)
XCS(I,J,IH) = C + SCF
XSS(I,J,IH) = C2 + SCF
YCS(I,J,IH) = C3 + SCF
504 YSS(I,J,IH) = C4 + SCF
IF(INDI4) 505,504,505
708 IF(I-J-KB) 504,708 + 505
709 WRIT-(7,504)J
710 WRITE(6,127)
711 WRITE(6,135)((CMR(I,K),K=1,KQ2),I=1,2)
712 WRITE(6,135)((CME(I,K),K=1,KQ2),I=1,2)
713 WRITE(6,135)J
714 WRITE(6,127)
715 WRITE(6,135)(XCS(I,J,IH),I=1,KQ2)
716 WRITE(6,135)(YCS(I,J,IH),I=1,KQ2)
717 WRITE(6,135)(XSS(I,J,IH),I=1,KQ2)
718 WRITE(6,135)(YSS(I,J,IH),I=1,KQ2)
505 CONTINUE
IF(INFI) 520,525,525
506 DO 507 I = 1, KQ2
507 DO 507 J = 1, KQ2
C1 = UR(I,J)
C2 = UE(I,J)
DO 506 K = 1, KQ2
C1 = C1 + WB(I,K)*S(K,J,IH) - HC(I,K)*SE(K,J,IH)
508 C2 = C2 + WB(I,K)*SE(K,J,IH) + HC(I,K)*SR(K,J,IH)
WR(I,J) = C1
509 WE(I,J) = C2
IF(KC) 511,508,510
508 C1 = WR(1,1)*WR(2,2) - WR(1,2)*WR(2,1) - WE(1,1)*WE(2,2) + WE(1,2)*WE(2,1)
C2 = WR(1,1)*WE(2,2) + WR(2,2)*WE(1,1) - WR(1,2)*WE(2,1) - WR(2,1)*WE(1,2)
C3 = C1 + C2
C1 = -C1/C3
C2 = -C2/C3
EMR(1,1) = C1*WR(2,2) + C2*WE(1,2)
EME(1,1) = C1*WE(2,2) - C2*WR(2,2)
EMR(1,2) = -C1*WR(1,1) - C2*WE(1,1)
EME(1,2) = C2*WR(1,1) - C1*WE(1,2)
EMR(2,1) = -C1*WR(2,1) - C2*WE(1,1)
EME(2,1) = C2*WR(2,1) - C1*WE(2,1)
EMR(2,2) = C1*WR(1,1) + C2*WE(1,1)
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      EME(2,2)=C *ME(1,1)-C*WR(1,1)
      GO TO 522
511 DO 512 I=1,4
      DO 512 J=1,4
      UR(I,J)=WR(I,J)
512 UE(I,J)=WE(I,J)
      CALL MATINV(UR,4,WCC,0,DVN,1D)
      GO TO (514,5,3),ID
513 WRITE(6,145)NF
      GO TO 530
514 CALL MATINV(UE,4,WCC,0,DVN,1D)
      GO TO (517,5,5),ID
515 DO 516 I=1,4
      DO 516 J=1,4
      EMR(I,J)=-UR(I,J)
516 EME(I,J)=0,0
      GO TO 522
517 DO 519 I=1,4
      DO 519 J=1,4
      C1=0,0
      DO 518 K=1,4
518 C1=C1+UR(I,K)*WE(K,J)+UE(I,K)*WR(K,J)
519 WA(I,J)=C.
      CALL MATINV(WA,4,WCC,0,DVN,1D)
      GO TO (710,709),ID
709 WRITE(6,145)NF
      GO TO 530
710 DO 521 I=1,4
      DO 521 J=1,4
      C1=0,0
      C2=0,0
      DO 520 K=1,4
520 C1=C1+WA(I,K)*UE(K,J)
      C2=C2-WA(I,K)*UR(K,J)
      EMR(I,J)=-C.
521 EME(I,J)=-C2
522 DO 524 I=1,KQ2
      DO 524 J=1,KQ2
      C1=0,0
      C2=0,0
      DO 523 K=1,KQ2
523 C1=C1+EMR(I,K)*WR(K,J)+EME(I,K)*WC(K,J)
      C2=C2+EME(I,K)*WB(K,J)-EMR(I,K)*WC(K,J)
      SR(I,J,NF)=C.
524 SE(I,J,NF)=C2
      GO TO 530
525 DO 528 I=1,KQ2
      C2=0,0
      DO 527 J=1,KQ2
      C1=0,0
      DO 526 K=1,KQ2
526 C1=C1+1.0*WB(I,K)*SR(K,J,1)-2.0*WC(I,K)*SE(K,J,1)
      C2=C2-C1*XZ(J)
527 DEVN(I,J)=C1+UR(I,J)
528 WCC(I,J)=C.
530 CONTINU-

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CALL MATINV(DEVN,KQ2,WCC,1,DVN,1D)          694
  GO TO 1540,531),ID
531 WRITE(6,133)
  GO TC 599
540 DO 541 I=1,KQ2
  XR(I,1)=WCC(I,1)
  XE(I,1)=0.0
541 XZ(I)=XZ(I)+WCC(I,1)
  DO 543 I=1,KQ2
  C1=0.0
  C2=0.0
  DO 542 J=1,KQ2
  C1=C1+2.0*SR(I,J,1)*XZ(J)
542 C2=C2+2.0*SE(I,J,1)*XZ(J)
  XR(I,2)=C1
543 XE(I,2)=C2
  WRITE(6,134)
  WRITE(6,139)
  NH1=NH1
  DO 553 IK=1,NH1
  NF=IK-1
  HN=NF
  FRQ=HN*SPD*SFR+
  LALM=0
  IF(INF-1) 550,550,545
545 DO 549 I=1,KQ2
  C1=0.0
  C2=0.0
  DO 546 K=1,KQ2
  C1=C1+SR(I,K,NF)*XR(K,NF)-SE(I,K,NF)*XE(K,NF)
546 C2=C2+SR(I,K,NF)*XE(K,NF)+SE(I,K,NF)*XR(K,NF)
  XR(I,IK)=C1
  IF(ABS(C1)-AMLM) 547,547,549
547 IF(ABS(C2)-AMLM) 548,548,549
548 LALM=LALM+1
549 CONTINUE
  IF(LALM-4) 550,544,544
544 NH2=IK
550 WRITE(1,14) INF,FRQ,XR(1,IK),XE(1,IK),XR(2,IK),XE(2,IK),XR(3,IK),
  ,XE(3,IK),XR(4,IK),XE(4,IK)          759
  DO 552 J=1,NS
  C1=0.0
  C2=0.0
  C3=0.0
  C4=0.0
  DO 551 K=1,KQ2
  C1=C1+XCS(K,J,IK)*XR(K,IK)-XSS(K,J,IK)*XE(K,IK)
  C2=C2+XCS(K,J,IK)*XE(K,IK)+XSS(K,J,IK)*XR(K,IK)
  C3=C3+YCS(K,J,IK)*XR(K,IK)-YSS(K,J,IK)*XE(K,IK)
551 C4=C4+YCS(K,J,IK)*XE(K,IK)+YSS(K,J,IK)*XR(K,IK)
  XCS(1,J,IK)=C1
  XSS(1,J,IK)=C2
  YCS(1,J,IK)=C3
552 YSS(1,J,IK)=C4
  IF(LALM-4) 553,557,559
553 CONTINUE

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SAL - EFN SOURCE STATEMENT - (EN(S)) -

07/25/67

539 DO 570 J=1,NS	
WRITE(6,131)J	803
WRITE(6,133)	804
DO 554 IK=1,NH2	
NF=IK-1	
HN=NF	
FRQ=HN*SPD*SFR	
554 WRITE(6,137)INF,FRQ,XCS(1,J,IK),XSS(1,J,IK),YCS(1,J,IK),YSS(1,J,IK)	808
WRITE(6,133)	813
WRITE(6,137)	814
C1=WTST	
555 C2=0.0,7453293*C1=SFR	
C3=XCS(1,J,1)	820
C4=YCS(1,J,1)	821
C5=0.0	
DO 556 IK=1,NH2	
C5=C5+C2	
C6=COS(C5)	
C7=SIN(C5)	
C3=C3+XCS(1,J,IK)*C6-XSS(1,J,IK)*C7	
556 C4=C4+YCS(1,J,IK)*C6-YSS(1,J,IK)*C7	
C5=SCRT(C3+C3+C4+C4)	828
D1=C3	
D2=C4	
IF(C3) 564,557,561	
557 IF(C4) 559,560,558	
558 C5=90.0	
GO TO 567	
559 C6=270.0	
GO TO 567	
560 C6=0.0	
GO TO 567	
561 IF(C4) 563,562,562	
562 C7=0.0	
C8=57.295780	
GO TO 566	
563 C7=360.0	
C8=-57.295780	
C4=-C4	
GO TO 566	
564 C3=-C3	
C7=180.0	
C8=-57.295780	
IF(C4) 565,566,566	
565 C8=-C8	
C4=-C4	
566 C6=C7+C8*ATAN(C4/C3)	848
567 WRITE(6,102)C1,D1,U2,C5,C6	849
C1=C1+WTIN	
IF(WTFN+0.00001-C1) 570,570,555	
570 CONTINUE	
599 SPD=SPD+SPIN	734
IF(SPFA+0.00001-SPD) 600,600,260	735
600 NSP1=NSP1+	736
IF(NSP-NSP1) 601,230,230	737
601 IF(INP) 602,190,612	738

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SAL - eFig SOURCE STATEMENT - IFN(S) -

132	FORMAT(P5L 4.5)	743
137	FORMAT(1, P5.5, .P7E.4, 0)	749
122	FORMAT(//,1H,HARMONIC NO.,13,1H,FREQUENCY=,1PE.3.5,8H RAD/SEC)	767
133	FORMAT(1H,HEATING COEFFICIENTS)	768
124	FORMAT(8H STATION) K3HKXX9X7HFRQ=8XX9X3HKXY9X7HFRQ=BXY9X3HKYY9X7HFR .Q=8YX9X3HKYY9X7HFRQ=BYY)	769
125	FORMAT(1H,ROTOR-EARING IMPEDANCE MATRIX)	770
127	FORMAT(1H,REAL PART)	771
126	FORMAT(1H,IMAGINARY PART)	
130	FORMAT(1.6H,REAL PART OF A-MATRIX IS SINGULAR)	
131	FORMAT(1H,HARMONIC=,13,1H FREQUENCY=,1PE13.6)	
132	FORMAT(1H,ROTOR STATION NO.,12)	
133	FORMAT(1.5H,FINAL MATRIX IS SINGULAR)	
134	FORMAT(4H,AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES)	
135	FORMAT(1H,HARMONIC FREQUENCY7X4HX(C),10X4HX(S)10X4HY(C)10X4HY(S))	
136	FORMAT(1H,WHIRL ORBITS)	
137	FORMAT(1H,SHAFT ROTAT,DEG5X.HX13X1HY9X9HAMPLITUDE4X12HANGLE X-AMP .L)	
138	FORMAT(1.4.5)	
139	FORMAT(1H,HARMONIC FREQUENCY5X4HX(C)8X4HX(S)8X4HY(C)8X4HY(S)10X8 .1HDX/DZ(C)4X6HDY/DZ(S)4X6HDY/DZ(C)4X6HDY/DZ(S))	
140	FORMAT(1.4H,REAL PART OF IMPEDANCE MATRIX IS SINGULAR)	
141	FORMAT(15, PE2.4, PE2.2, 4)	
142	FORMAT(1.7H,REAL PART OF S-MATRIX IS SINGULAR,K=,13)	
143	FORMAT(1H,INVERSE IMPEDANCE MATRIX)	
144	FORMAT(5+H,INFLUENCE COEFFICIENT MATRIX E (X(J)=E+F) FOR STATION,I .3)	
145	FORMAT(-6H,INFLUENCE MATRIX A (X(J)=A*X(KB)) FOR STATION,I3)	
146	FORMAT(1.5H,INVERSION MATRIX FOR A IS SINGULAR)	
147	FORMAT(1.5H,INVERSION MATRIX FOR E IS SINGULAR)	
148	FORMAT(1.8H,INVERSION MATRIX FOR S IS SINGULAR,K=13)	
160	RETURN	
130	Rc TURN	
190	RETURN	
602	RETURN	
	END	776

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SUBROUTINE BB2
COMMON/A/ BKXX(10,11),BCXX(10,11),BKXY(10,11),BCXY(10,11),
1BKYL(10,11),BCYX(10,11),BKYY(10,11),BCYY(10,11),PMEN(0,5),
2PADM(10,5),PADK(10,5),PANG(10,5),DEVN(4,4),WCC(4,4),CPK(4,8),
3CME(4,8),AMR(4,8),AME(4,8),WR(4,4),WE(4,4),WA(4,4),WB(4,4),
4WC(4,4),WQ(4,4),WSG(4,4),UR(4,4),UE(4,4),EMR(4,4),EMF(4,4),
5XR(4,11),XC(4,11)
COMMON/B/ PKXX(10,5,11),PCXX(10,5,11),PKXY(10,5,11),PCXY(10,5,11),
1PKYX(10,5,11),PCYX(10,5,11),PKYY(10,5,11),PCYY(10,5,11),
2GR(2,8,50),GE(2,8,50),SR(4,4,-1),SE(4,4,-1),XCS(4,50,-1),
3XSS(4,50,1),YCS(4,50,11),YSS(4,50,-1)
COMMON/C/ RM(30),RIP(30),RIT(30),RS(30),RW(30),RD(30),RL(30),
1DVXA(30),DVXB(30),DVXC(30),DVXD(30),DVYA(30),DVYB(30),DVYC(30),
2DVYD(30),DVUX(30),DVUY(30),DMUX(30),DMUY(30),DMXA(30),BL(30),
3BZ(30),B3(30),B4(30),B5(30),BL(30),B7(30),B8(30),B9(30),B10(30),
4PMX(10),PKX(10),PDX(10),PHY(10),PKY(10),PDY(10),SXX(10),DXX(10),
5SXV(10),DXY(10),SYX(10),DYX(10),SYY(10),DYY(10),LB(10),
6XZ(4),XZ(4)
COMMON/D/A_,A2,A3,A4,A5,AS,A7,A8,NF,FRQ,KC,SCFI,NDIA,KQ2,NS,K8,
1IK,NH2,HN,AMLM,SPD,SFRI,SFR,WTST,WTIN,WTF, NH1
COMMON/E/KQ1,KQ3,C1,C2,N8,KA,NRP,NPD,INC,NH,NSP,INP,YF,DNST,SHM,
*SPST,SPFN,SPIN,SCF,QZ,QZP,K1,K2, WTN
COMMON/F/BMXC,BMXS,BMYC,BMYS,VXC,VXS,VYC,VYS,XC,XS,YC,YS,DXC,DXS,
*DYS,C3,C4, NS1, NPD1, NPD2, DYC, NSPI
DO 485 I=1,K03
BMXC=0.0
BMXS=0.0
BMYC=0.0
BMYS=0.0
VXC=0.0
VXS=0.0
VYC=0.0
VYS=0.0
XC=0.0
XS=0.0
YC=0.0
YS=0.0
DXC=0.0
DXS=0.0
DYC=0.0
DYS=0.0
DVUX(KB)=0.0
DVUY(KB)=0.0
DMUX(KA)=0.0
DMUY(KB)=0.0
GO TO 451,462,463,464,465,469,468,472,I
XC=C,CC,
GO TO 475
YC=C,CC,
GO TO 475
DXC=0.0,
GU TO 475
DYC=0.0,
GU TO 475
IF(KC) -457,465,+cc

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465 IF(KA) 453,467,467	
467 DVUX(KB)=...	
GO TO 472	
468 DMUX(KB)=...	
GO TO 473	
469 IF(KC) 471,471,471	
470 IF(KA) 471,471,471	
471 DVUY(KB)=...	
GU TO 475	
472 DMUY(KB)=...	
473 DO 480 J=1,NS	
C1=DMXA(J)	421
C2=FRQ*SPD*KIP(J)	422
GR(1,1,J)=XC	423
GR(2,1,J)=YC	
GE(1,1,J)=XS	
Ge(2,1,J)=YS	
IF(J-KB) 477,476,477	
476 CMR(1,1)=XC	424
CME(1,1)=XS	425
CMR(2,1)=YC	426
CME(2,1)=YS	427
CMR(3,1)=DAX	428
CME(3,1)=DXS	429
CMR(4,1)=DYC	430
CME(4,1)=DYS	431
477 A1=BMXC-C1*DAX-C2*DYS-DMUX(J)	432
A2=BMXS-C1*DAX+C2=DYC	433
A3=BMYC-C1*DYC+C2=DAX-DMUY(J)	434
A4=BMYS-C1*DYS-C2=DXC	435
A5=VXC+DVXA(J)*XC+DVXB(J)*XS-DVXC(J)*YC+DVXD(J)*YS+DVUX(J)	436
A6=VXS-DVXB(J)*XC+DVXA(J)*XS-DVXD(J)*YC-DVXC(J)*YS	437
A7=VYC-DVYC(J)*XC+DVYD(J)*XS+DVYA(J)*YC+DVYB(J)*YS+DVUY(J)	438
A8=VYS-DVYD(J)*XC-DVYC(J)*XL-DVYB(J)*YC+DVYA(J)*YS	439
IF(NS-J) 480,480,-7H	440
478 C1=XC	441
C2=XS	442
C3=YC	443
C4=YS	444
BMXC=C1*B9(J)+DXC*B10(J)+A1*B2(J)+A5*B3(J)	445
BMXS=C2*B9(J)+DXS*B10(J)+A2*B2(J)+A6*B3(J)	446
BMYC=C3*B9(J)+DYC*B10(J)+A3*B2(J)+A7*B3(J)	447
BMYS=C4*B9(J)+DYS*B10(J)+A4*B2(J)+A8*B3(J)	448
VXC=C1*B6(J)+DXC*B9(J)+A1*B5(J)+A5*B6(J)	449
VXS=C2*B6(J)+DXS*B9(J)+A2*B5(J)+A6*B6(J)	450
VYC=C3*B6(J)+DYC*B9(J)+A3*B5(J)+A7*B6(J)	451
VYS=C4*B6(J)+DYS*B9(J)+A4*B5(J)+A8*B6(J)	452
XC=C1*B1(J)+DXC*B1(J)+A1*B1(J)+A5*B7(J)	453
XS=C2*B1(J)+DXS*B1(J)+A2*B1(J)+A6*B7(J)	454
YC=C3*B1(J)+DYC*B1(J)+A3*B4(J)+A7*B7(J)	455
YS=C4*B1(J)+DYS*B1(J)+A4*B4(J)+A8*B7(J)	456
DXC=C1*B5(J)+DXC*B1(J)+A1*B1(J)+A5*B4(J)	457
DXS=C2*B5(J)+DXS*B1(J)+A2*B1(J)+A6*B4(J)	458
DYC=C3*B5(J)+DYC*B1(J)+A3*B1(J)+A7*B4(J)	459
DYS=C4*B5(J)+DYS*B1(J)+A4*B1(J)+A8*B4(J)	460
480 CONTINUE	461
	462

LASS

- EPN STATEMENT - IIN(S) -

27/25/67

AMR(1,I)=A1
AME(1,I)=A2
AMR(2,I)=A3
AME(2,I)=A4
AMR(3,I)=A5
AME(3,I)=A6
AMR(4,I)=A7
AME(4,I)=A8

485 CONTINUE
RETURN
END

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C MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS      MAT1   4
C NOVEMBER 1962  S GOOD DAVID TAYLOR MODEL BASIN AM MAT.      MAT2   5
C
C SUBROUTINE MATINV(A,N,B,M,DETERM,ID)      MAT3   6
C
C GENERAL FORM OF DIMENSION STATEMENT      MAT4   7
C
C DIMENSION A(4,4),B(4,1)      MAT5   8
C DIMENSION INDEX(4,3)      MAT6   9
C EQUIVALENCE (IROW,JROW), (ICOLUMN,JCOLUMN), (AMAX, T, SWAP)      MAT7   10
C
C INITIALIZATION      MAT8   11
C
C M=M;
C N=N;
C DO 8 I=1,N      MAT9   12
C K1=1      MAT10  13
C K2=1      MAT11  14
C DO 6 J=1,N      MAT12  15
C IF(A(I,J)) 3,4,3      MAT13  16
C 3 K1=0      MAT14  17
C 4 IF(A(J,I)) 5,6,5      MAT15  18
C 5 K2=0      MAT16  19
C 6 CONTINUE      MAT17  20
C IF(K1+K2) 8,8,7      MAT18  21
C 7 ID=2      MAT19  22
C DETERM=0.0      MAT20  23
C GO TO 740      MAT21  24
C 8 CONTINUE      MAT22  25
C 10 DETERM=.0      MAT23  26
C 15 DO 20 J=1,N      MAT24  27
C 20 INDEX(J,3)=0      MAT25  28
C 30 DO 550 I=1,N      MAT26  29
C
C SEARCH FOR PIVOT ELEMENT      MAT27  30
C
C 40 AMAX=0.0      MAT28  31
C 45 DO 105 J=1,N      MAT29  32
C IF(INDEX(J,3)=1) 50, 105, 60      MAT30  33
C 60 DO 100 K=1,N      MAT31  34
C IF(INDEX(K,3)=1) 80, 100, 745      MAT32  35
C 65 IF(AMAX-ABS(A(J,K))) 85,100,.00      MAT33  36
C 85 IROW=J      MAT34  37
C 90 ICOLUMN=K      MAT35  38
C AMAX=ABS(A(J,K))      MAT36  39
C 100 CONTINUE      MAT37  40
C 105 CONTINUE      MAT38  41
C     INDEX(ICOLUMN,3)=INDEX(ICOLUMN,3)+.      MAT39  42
C 260 INDEX(I,1)=IROW
C 270 INDEX(I,2)=ICOLUMN
C
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
C 130 IF (IROW-ICOLUMN) 140, 310, 140
C 140 DETERM=-DETERM

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DAVE - EPN SOURCE STATEMENT - IFN(1) -

07/29/67

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150 DO 200 L=1,M
160 SWAP=A(IFORW,L)
170 A(IFORW,L)=A(ICOLUMN,L)
200 A(ICOLUMN,L)=SWAP
    IF(M) 3.0V 3.0, 2.0
210 DO 250 L=1,M
220 SWAP=B(IFORW,L)
230 B(IFORW,L)=B(ICOLUMN,L)
250 B(ICOLUMN,L)=SWAP

C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
310 PIVOT =A(ICOLUMN,ICOLUMN)
DETERM=DETERM*PIVOT
330 A(ICOLUMN,ICOLUMN)=1.0
340 DO 350 L=1,N
350 A(ICOLUMN,L)=A(ICOLUMN,L)/PIVOT
355 IF(M) 380, 380, 360
360 DO 370 L=1,M
370 B(ICOLUMN,L)=B(ICOLUMN,L)/PIVOT

C      REDUCE NON-PIVOT ROWS
C
380 DO 550 L=1,N
390 IF(L1=ICOLUMN) 400, 550, 400
400 T=A(L1,ICOLUMN)
420 A(L1,ICOLUMN)=0.0
430 DO 450 L=1,N
450 A(L1,L)=A(L1,L)-A(ICOLUMN,L)*T
455 IF(M) 550, 550, 460
460 DO 500 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLUMN,L)*T
550 CONTINUE

C      INTERCHANGE COLUMNS
C
600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)=INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUMN=INDEX(L,2)
650 DO 700 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUMN)
700 A(K,JCOLUMN)=SWAP
700 CONTINUE
710 CONTINUE
    DO 730 K=1,N
    IF(INDEX(K,3)=1) 7.5, 720, 7.5
720 CONTINUE
730 CONTINUE
    ID=_
C      LAST CARD OF PROGRAM
740 RETURN
745 ID=_
    GO TO 740

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HAT1	43
HAT1	44
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HAT1	89
HAT1	90
HAT1	91
HAT1	94
HAT1	95
HAT1	96
HAT1	96
HAT1	97
HAT1	97
HAT1	98
HAT1	98

TURBINE DRIVEN 4 POLE ALTERNATOR RESPONSE 4-11-1967

11	2	7	-1	0	4	1	5	2	0	1
300000000.0	0.283				86300000.0					
6.15	62.4	31.0			1.3	3.5		3.5		3.339
1.61	0.0	0.0			1.45	3.5		3.6		3.2
0.0	0.0	0.0			2.37	3.5		3.65		3.062
0.0	0.0	0.0			4.83	3.5		3.65		3.062
0.0	0.0	0.0			1.67	2.775		3.25		2.0
3.1	12.0	12.0			2.1	3.5		3.7		1.04
0.0	0.0	0.0			2.1	3.5		3.7		1.04
3.1	12.0	12.0			1.67	2.775		3.25		2.0
0.0	0.0	0.0			3.49	3.5		3.65		3.062
0.0	0.0	0.0			4.39	3.5		3.65		3.062
9.21	56.0	28.0			0.0	1.0		0.0		0.0
4	10									
0.000000E+00	3.400000E+02	2.000000E+01	1.000000E-20							
8000.0	21971.0	4657.0	2.0							1.0
20000.0	88200.0									
0.0	0.0	42000.0	0.0							
0.0	0.0	0.0	-42000.0							
42000.0	0.0	0.0	0.0							
0.0	-42000.0	0.0	0.0							
0.0	0.0	0.0	-42000.0							
0.0	0.0	-42000.0	0.0							
-42000.0	0.0	0.0	0.0							
0.0	0.0	0.002	0.0							
0.13	0.57	300000.0	45.0							
39600.0	25560.0	19550.0	-19700.0	2455.0	-605.0	2720.0	1202.0			
63000.0	7360.0	1470.0	-4810.0	2020.0	-1023.0	3710.0	1377.0			
68300.0	2340.0	-1227.0	-1202.0	268.5	-677.0	5875.0	843.0			
70300.0	1190.0	-1650.0	-575.0	-677.0	-415.0	7060.0	524.0			
71200.0	742.0	-1828.0	-352.0	-1202.0	-281.0	7700.0	352.0			
71900.0	537.0	-1937.0	-241.5	-1558.0	-203.0	8130.0	261.0			
0.13	0.57	400000.0	135.0							
162500.0	86200.0	66500.0	-94100.0	9480.0	2440.0	14350.0	-1970.0			
229500.0	7480.0	-4270.0	-6810.0	7610.0	-3910.0	17770.0	4370.0			
226500.0	1435.0	735.0	-1705.0	2970.0	-1370.0	22500.0	1203.0			
224500.0	534.0	969.0	-869.0	1435.0	-745.0	23400.0	607.0			
224000.0	274.0	434.0	-544.0	701.0	-484.0	23800.0	378.0			
223500.0	160.5	0.0	-384.0	277.0	-348.0	24050.0	261.0			
0.13	0.57	300000.0	45.0							
39600.0	25560.0	19550.0	-19700.0	2455.0	-605.0	2720.0	1202.0			
63000.0	7360.0	1470.0	-4810.0	2020.0	-1023.0	3710.0	1377.0			
68300.0	2340.0	-1227.0	-1202.0	268.5	-677.0	5875.0	843.0			
70300.0	1190.0	-1650.0	-575.0	-677.0	-415.0	7060.0	524.0			
71200.0	742.0	-1828.0	-352.0	-1202.0	-281.0	7700.0	352.0			
71900.0	537.0	-1937.0	-241.5	-1558.0	-203.0	8130.0	261.0			
0.13	0.57	400000.0	135.0							
162500.0	86200.0	66500.0	-94100.0	9480.0	2440.0	14350.0	-1970.0			
229500.0	7480.0	-4270.0	-6810.0	7610.0	-3910.0	17770.0	4370.0			
226500.0	1435.0	735.0	-1705.0	2970.0	-1370.0	22500.0	1203.0			
224500.0	534.0	969.0	-869.0	1435.0	-745.0	23400.0	607.0			
224000.0	274.0	434.0	-544.0	701.0	-484.0	23800.0	378.0			
223500.0	160.5	0.0	-384.0	277.0	-348.0	24050.0	261.0			
7757.0	9557.00	300.0		2.0						
20000.0	88200.0									
0.0	0.0	42000.0	0.0							
0.0	0.0	0.0	-42000.0							
42000.0	0.0	0.0	0.0							
0.0	-42000.0	0.0	0.0							

0.0	0.0	0.0	-42000.0				
0.0	0.0	-42000.0	0.0				
0.0	-42000.0	0.0	0.0				
-42000.0	0.0	0.0	0.0				
0.0	0.0	0.002	0.0				
0.13	0.57	300000.0	45.0				
39600.0	25560.0	19550.0	-19700.0	2455.0	-605.0	2720.0	1202.0
63000.0	7360.0	1470.0	-4810.0	2020.0	-1023.0	3710.0	1377.0
68300.0	2340.0	-1227.0	-1202.0	268.5	-677.0	5875.0	843.0
70300.0	1190.0	-1650.0	-575.0	-677.0	-415.0	7060.0	524.0
71200.0	742.0	-1828.0	-352.0	-1202.0	-281.0	7700.0	352.0
71900.0	537.0	-1937.0	-241.5	-1558.0	-203.0	8130.0	261.0
0.13	0.57	400000.0	135.0				
162500.0	86200.0	66500.0	-94100.0	9480.0	2440.0	14350.0	-1970.0
229500.0	7480.0	-4270.0	-6810.0	7610.0	-3910.0	17770.0	4370.0
226500.0	1435.0	735.0	-1705.0	2970.0	-1370.0	22500.0	1203.0
224500.0	534.0	969.0	-869.0	1435.0	-745.0	23400.0	607.0
224000.0	274.0	434.0	-544.0	701.0	-484.0	23800.0	378.0
223500.0	160.5	0.0	-384.0	277.0	-348.0	24050.0	261.0
0.13	0.57	300000.0	45.0				
39600.0	25560.0	19550.0	-19700.0	2455.0	-605.0	2720.0	1202.0
63000.0	7360.0	1470.0	-4810.0	2020.0	-1023.0	3710.0	1377.0
68300.0	2340.0	-1227.0	-1202.0	268.5	-677.0	5875.0	843.0
70300.0	1190.0	-1650.0	-575.0	-677.0	-415.0	7060.0	524.0
71200.0	742.0	-1828.0	-352.0	-1202.0	-281.0	7700.0	352.0
71900.0	537.0	-1937.0	-241.5	-1558.0	-203.0	8130.0	261.0
0.13	0.57	400000.0	135.0				
162500.0	86200.0	66500.0	-94100.0	9480.0	2440.0	14350.0	-1970.0
229500.0	7480.0	-4270.0	-6810.0	7610.0	-3910.0	17770.0	4370.0
226500.0	1435.0	735.0	-1705.0	2970.0	-1370.0	22500.0	1203.0
224500.0	534.0	969.0	-869.0	1435.0	-745.0	23400.0	607.0
224000.0	274.0	434.0	-544.0	701.0	-484.0	23800.0	378.0
223500.0	160.5	0.0	-384.0	277.0	-348.0	24050.0	261.0

TURBINE DRIVEN 4 POLE ALTERNATOR RESPONSE 4-21-1967

STATIONS	BEARINGS	MAGN. ST	F+M/F/M	RIG.PED	NO.PADS	COMPRESS	HARMONICS	NO.SPEEDS	DIA(GS)	INPUT
11	2	7	-1	0	4	1	5	2	0	1

YOUNGS MOD. DENSITY (SHAPE FACT) G
 3.00000E 07 2.03000E-01 8.63000E 06

ROTOR DATA

STATION	MASS, LBS	PULAR MOM. IN.	TRANSY. MOM. IN	LENGTH	OUT. BLASTPIPE	OUT.DIA(MASS)	INNER DIA.
1	6.15000E 00	6.24000E 04	3.10000E 01	1.30000E 00	3.50000E 00	3.33900E 00	
2	1.61000E 00	0.	0.	1.45000E 00	3.90000E 00	3.20000E 00	
3	0.	0.	0.	2.37000E 00	3.90000E 00	3.06200E 00	
4	0.	0.	0.	4.83000E 00	3.90000E 00	3.65000E 00	
5	0.	0.	0.	1.67000E 00	2.77500E 00	3.25000E 00	
6	3.10000E 00	1.20000E 01	1.20000E 01	2.10000E 00	3.50000E 00	3.70000E 00	1.04000E 00
7	0.	0.	0.	2.10000E 00	3.50000E 03	3.70000E 00	1.04000E 00
8	3.10000E 00	1.20000E 01	1.20000E 01	1.67000E 00	2.77500E 00	3.25000E 00	2.00000E 00
9	0.	0.	0.	3.49000E 00	3.50000E 00	3.65000E 00	3.06200E 00
10	0.	0.	0.	4.39000E 00	3.50000E 00	3.65000E 00	3.06200E 00
11	9.21000E 00	5.60000E 01	2.80000E 01	0.	1.00000E 00	0.	0.

BEARING STATIONS

4	10
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MAGNETIC FORCE DATA

0.01	FORCE	Q101.MOMENT
2.000000E 04	8.820000E 04	

MATRIX OF COSINE COMPONENTS

0.	0.	4.200000E 04	0.
0.	0.	0.	-4.200000E 04
0.	-4.200000E 04	0.	0.
0.	-4.200000E 04	0.	0.

MATRIX OF SINE COMPONENTS

0.	0.	0.	-2.000000E 04
0.	0.	-4.200000E 04	0.
0.	-4.200000E 04	0.	0.
0.	-4.200000E 04	0.	0.

COORDINATES OF STATIC ROTOR ECCENTRICITY
0.
0.
2.000000E-03 0.

BEARING DATA

BEARINGS AT STATION 4

PAD NO. 1
PITCH MOM. IN. 1.300000E-01 PAD MASS 5.700000E-01 PIVOT STIFFN. 3.000000E-03 PIVOT ANGLE 4.500000E-01

HARMONIC	KXX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY
0	3.960000E 04	2.560000E 04	1.955000E 04	-1.970000E 04	2.4555000E 03	-6.050000E 02	2.720000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03
1	6.300000E 04	7.360000E 03	1.400000E 03	-4.000000E 03	2.020000E 03	-1.023000E 03	3.710000E 03	1.3717000E 03						
2	1.250000E 04	2.340000E 03	-1.227000E 03	-1.220000E 03	2.683000E 03	-6.700000E 02	5.875000E 03	8.436000E 02						
3	7.030000E 04	1.730000E 03	-1.630000E 03	-5.750000E 03	-6.770000E 02	-1.150000E 02	7.050000E 03	3.240000E 02						
4	7.120000E 04	7.420000E 02	-1.628000E 02	-1.520000E 02	-1.202000E 02	-2.810000E 02	7.700000E 03	3.240000E 02						
5	7.190000E 04	5.370000E 02	-1.937000E 03	-2.455000E 02	-1.555000E 03	-2.030000E 02	8.130000E 03	2.610000E 02						

PAD NO. 2
PITCH MOM. IN. 1.300000E-01 PAD MASS 5.700000E-01 PIVOT STIFFN. 3.000000E-03 PIVOT ANGLE 1.330000E-02

HARMONIC	KXX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY
0	1.625000E 05	8.620000E 04	6.450000E 04	-9.410000E 04	9.460000E 03	2.440000E 03	1.435000E 03	-1.435000E 03	1.435000E 03	-1.435000E 03	1.435000E 03	1.435000E 03	1.435000E 03	1.435000E 03
1	2.795000E 05	7.480000E 03	-8.270000E 03	-6.810000E 03	7.610000E 03	-3.910000E 03	1.777000E 03	-1.777000E 03	1.777000E 03	-1.777000E 03	1.777000E 03	1.777000E 03	1.777000E 03	1.777000E 03
2	2.265000E 05	1.435000E 03	7.350000E 02	-1.705000E 03	2.910000E 03	-1.310000E 03	2.250000E 03	-1.250000E 03	2.250000E 03	-1.250000E 03	2.250000E 03	2.250000E 03	2.250000E 03	2.250000E 03
3	2.245000E 05	5.340000E 02	9.890000E 02	-8.690000E 02	1.335000E 02	-7.430000E 02	2.340000E 02	-6.700000E 02	2.340000E 02	-6.700000E 02	2.340000E 02	2.340000E 02	2.340000E 02	2.340000E 02
4	2.240000E 05	2.740000E 02	6.300000E 02	-5.440000E 02	1.010000E 02	-4.040000E 02	2.360000E 02	-8.760000E 02	2.360000E 02	-8.760000E 02	2.360000E 02	2.360000E 02	2.360000E 02	2.360000E 02
5	2.235000E 05	1.603000E 02	0.	-3.460000E 02	2.770000E 02	-3.460000E 02	2.460000E 02	-2.460000E 02	2.460000E 02	-2.460000E 02	2.460000E 02	2.460000E 02	2.460000E 02	2.460000E 02

BEARING AT STATION 10

PAD NO. 1
PITCH MOM. IN. 1.300000E-01 PAD MASS 5.700000E-01 PIVOT STIFFN. 3.000000E-03 PIVOT ANGLE 1.330000E-02

HARMONIC	KXX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY
0	3.960000E 04	2.550000E 04	1.955000E 04	-1.970000E 04	2.4555000E 03	-6.050000E 02	2.720000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03	1.262000E 03
1	6.300000E 04	7.360000E 03	1.400000E 03	-4.000000E 03	2.020000E 03	-1.023000E 03	3.710000E 03	1.3717000E 03						
2	6.830000E 04	3.340000E 03	-2.220000E 03	-1.202000E 03	2.683000E 03	-6.700000E 02	5.875000E 03	8.436000E 02						
3	7.030000E 04	1.195000E 03	-1.650000E 03	-5.750000E 03	-6.770000E 02	-1.150000E 02	7.050000E 03	3.240000E 02						
4	7.120000E 04	7.620000E 02	-1.820000E 03	-3.520000E 02	-1.202000E 03	-2.810000E 02	7.700000E 03	3.240000E 02						

5 7.19000E 04 5.37000E 02 -1.93700E 03 -2.41500E 02 -1.55800E 03 -2.03000E 02 0.13000E 03 2.61000E 02
 PAD NO. 2
 PITCH MOM. IN. 2
 PAD MASS 1.30000E-01
 PIVOT STIFFN. 5.70000E-01
 PIVOT ANGLE 1.35000E 02

HARMONIC	KXX	KYY	KXY	KYX	KYY	KXY	KYX	KYY	KXY	KYX	KYY
0	1.62500E 05	8.62000E 04	6.65000E 04	-9.40000E 04	9.48000E 03	2.44000E 03	1.43500E 03	1.43500E 03	2.44000E 03	1.43500E 03	1.43500E 03
1	2.29500E 05	7.48000E 03	-4.27000E 03	-6.81000E 03	7.61000E 03	-3.91000E 03	1.77700E 03	1.77700E 03	-3.91000E 03	1.77700E 03	-1.97000E 03
2	2.26500E 05	1.43500E 03	7.35000E 02	-1.70500E 02	2.97000E 03	-1.37000E 03	2.25000E 03	2.25000E 03	-1.37000E 03	2.25000E 03	4.37000E 03
3	2.24500E 05	5.34000E 02	9.69000E 02	-1.69000E 02	1.43500E 02	1.43500E 02	-7.45000E 02	2.34000E 02	1.43500E 02	-7.45000E 02	1.20300E 03
4	2.24000E 05	2.74000E 02	6.34000E 02	-5.44000E 02	7.01000E 02	7.01000E 02	-1.84000E 02	2.38000E 02	7.01000E 02	-1.84000E 02	3.78000E 02
5	2.23500E 05	1.60500E 02	0.	-3.84000E 02	2.77000E 02	-3.84000E 02	2.77000E 02	-3.84000E 02	2.77000E 02	-3.84000E 02	2.40500E 02

ROTOR SPEED= 8.000000E 03 RPM

AMPLITUDES AT ROTOR STATION WITH MAGNETIC FORCES

HARMONIC	FREQUENCY	X(C)	Y(C)	X(S)	Y(S)	DX/DZ(C)	DY/DZ(C)	DX/DZ(S)	DY/DZ(S)
0	0.	-3.4861E-05	0.	4.4233E-06	0.	-4.1316E-06	0.	4.2108E-06	0.
1	1.6755E 03	1.0993E-03	1.1696E-03	1.1696E-03	-1.0993E-03	1.3287E-04	1.1032E-05	1.1032E-05	-1.3287E-04
2	3.3511E 03	-2.7853E-12	0.	4.2930E-12	0.	1.5810E-13	0.	-1.7885E-13	0.
3	5.0265E 03	4.4291E-15	0.	1.1720E-14	0.	8.0273E-16	0.	1.1315E-15	0.
4	6.7021E 03	5.5666E-18	0.	-6.4772E-18	0.	-2.0595E-18	0.	5.2307E-18	0.
5	8.3776E 03	-9.8293E-21	0.	-2.5198E-20	0.	-2.8520E-21	0.	-2.4617E-21	0.

ROTOR STATION NO. 1
HARMONIC FREQUENCY

	X(C)	X(S)	V(C)	V(S)
0	0.	2.158623E-05	-0.	-4.207629E-03 -0.
1	1.575516E 03	-7.658826E-04	9.240380E-04	9.240380E-04 7.658811E-04
2	3.354032E 03	4.883042E-12	-7.188894E-12	-1.031933E-11 -3.611343E-12
3	5.026548E 03	1.389998E-13	-1.672908E-13	6.110268E-16 7.691916E-17
4	6.702065E 03	-1.769759E-18	1.304734E-17	4.763008E-18 6.314209E-18
5	8.377581E 03	-8.687957E-22	-3.184666E-21	7.643583E-21 6.913349E-22

WHIRL ORBIT
SHAFT ROTAT, DEG

	X	Y	AMPLITUDE	ANGLE X-ANPL
0.	-7.442964E-04	8.819608E-04	1.154050E-03	1.301614E 02
2.000000E 01	-1.159074E-03	1.734788E-04	1.171984E-03	1.714877E 02
4.000000E 01	-1.021408E-03	-6.358629E-04	1.203160E-03	2.19037E 02
6.000000E 01	-3.957128E-04	-1.167364E-03	1.232610E-03	2.512744E 02
8.000000E 01	4.252409E-04	-1.172330E-03	1.247072E-03	2.899373E 02
1.000000E 02	1.057320E-03	-6.484364E-04	1.240321E-03	3.284799E 02
1.200000E 02	1.204768E-03	1.591807E-04	1.215238E-03	7.526646E 00
1.400000E 02	7.985917E-04	8.726284E-04	1.182890E-03	4.753660E 01
1.600000E 02	2.684613E-05	1.158076E-03	1.158436E-03	8.857313E 01
1.800000E 02	-7.442965E-04	8.819607E-04	1.134050E-03	1.301614E 02
2.000000E 02	-1.159074E-03	1.734786E-04	1.171984E-03	1.714877E 02
2.200000E 02	-1.021408E-03	-6.358630E-04	1.203160E-03	2.19038E 02
2.400000E 02	-3.957126E-04	-1.167364E-03	1.232610E-03	2.512744E 02
2.600000E 02	4.252411E-04	-1.172330E-03	1.247072E-03	2.899373E 02
2.800000E 02	1.057320E-03	-6.484362E-04	1.240321E-03	3.284799E 02
3.000000E 02	1.204768E-03	1.591810E-04	1.215238E-03	7.526657E 00
3.200000E 02	7.985916E-04	8.726285E-04	1.182890E-03	4.753661E 01

ROTOR STATION NO. 2
HARMONIC FREQUENCY

	X(C)	X(S)	Y(C)	Y(S)
0 0.	1.624258E-05	-0.	-3.789821E-05	-0.
1 1.679516E 03	-5.935989E-04	9.294029E-04	9.294031E-04	5.935975E-04
2 3.351032E 03	3.538685E-12	-6.459052E-12	-7.625767E-12	-3.244694E-12
3 5.026548E 03	1.238280E-15	-1.845368E-15	1.875511E-15	1.553076E-17
4 6.702065E 03	6.462117E-18	2.093138E-17	-1.120950E-17	1.070200E-17
5 8.377581E 03	-2.991946E-22	-1.151740E-20	1.193818E-20	7.67624E-22

WHIRL ORBIT

SHAFT ROTAT, DEG	X	Y	AMPLITUDE	ANGLE X-AMPL
0.	-5.773583E-04	8.915018E-04	1.062128E-03	1.229280E 02
2.000000E 01	-1.035889E-03	2.925064E-04	1.076395E-03	1.642318E 02
4.000000E 01	-1.002118E-03	-4.610891E-04	1.103106E-03	2.047078E 02
6.000000E 01	-4.918444E-04	-1.016669E-03	1.129392E-03	2.441832E 02
8.000000E 01	2.561686E-04	-1.114271E-03	1.143338E-03	2.829472E 02
1.000000E 02	8.919176E-04	-7.082262E-04	1.138904E-03	3.215487E 02
1.200000E 02	1.117928E-03	1.147245E-05	1.117987E-03	5.879621E-01
1.400000E 02	8.284482E-04	7.080700E-04	1.089812E-03	4.052032E 01
1.600000E 02	1.589279E-04	1.055621E-03	1.067517E-03	8.143819E 01
1.800000E 02	-5.773564E-04	8.915017E-04	1.062128E-03	1.229280E 02
2.000000E 02	-1.035889E-03	2.925062E-04	1.076395E-03	1.642318E 02
2.200000E 02	-1.002118E-03	-4.610892E-04	1.103106E-03	2.047079E 02
2.400000E 02	-4.918442E-04	-1.016669E-03	1.129392E-03	2.441832E 02
2.600000E 02	2.561688E-04	-1.114271E-03	1.143338E-03	2.829472E 02
2.800000E 02	8.919177E-04	-7.082260E-04	1.138904E-03	3.215487E 02
3.000000E 02	1.117928E-03	1.147263E-05	1.117987E-03	5.879718E-01
3.200000E 02	8.284481E-04	7.080701E-04	1.089812E-03	4.052033E 01

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Stability and Response of an Alternator Rotor

⑨

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ABSTRACT

This volume presents an analytical investigation of the vibrations induced in an alternator rotor by the generated electromagnetic forces. Formulas are given from which the magnetic forces can be calculated for three brushless alternator types: (1) the homopolar generator, (2) the heteropolar inductor generator, and (3) the two-coil Lundell generator. Numerical examples are given to illustrate the practical use of the formulas.

Two computer programs have been written for evaluation of the effect of the magnetic forces on the rotor. Manuals are provided for both programs, containing listings of the programs and giving detailed instructions for preparation of input data and for performing the calculations. The first computer program examines the stability of the rotor and the second program calculates the amplitude response of the rotor resulting from a built-in eccentricity and misalignment between the axes of the rotor and the alternator stator. Sample calculations are provided.

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